SLOVAK UNIVERSITY OF TECHNOLOGY IN BRATISLAVA

European Space Agency
Agence spatiale européenne

Chemical Propulsion Systems

Vladimír Kutiš, Pavol Valko

Space for Education, Education for Space
ESA Contract No. 4000117400/16NL/NDe

Specialized lectures

Contents

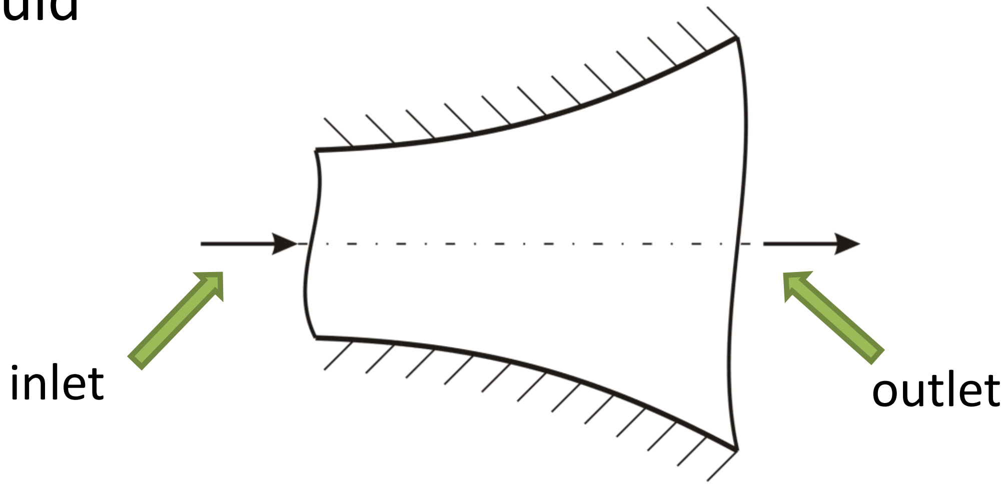
1. Fluid Flow and Thermodynamics

- Introduction
- Fundamental equations
- Thermodynamics of gases
- Speed of sound
- Isentropic flow
- Nozzle fluid flow

1. Fluid Flow and Thermodynamics

- Fluid flow:
  - naturally three dimensional, but in some special cases can be considered as one dimensional or quasi-one dimension
  - fluid can be considered according to:
    - steady-state VS transient
    - turbulent VS laminar
    - inviscid VS viscous fluid

quasi-one dimension fluid flow

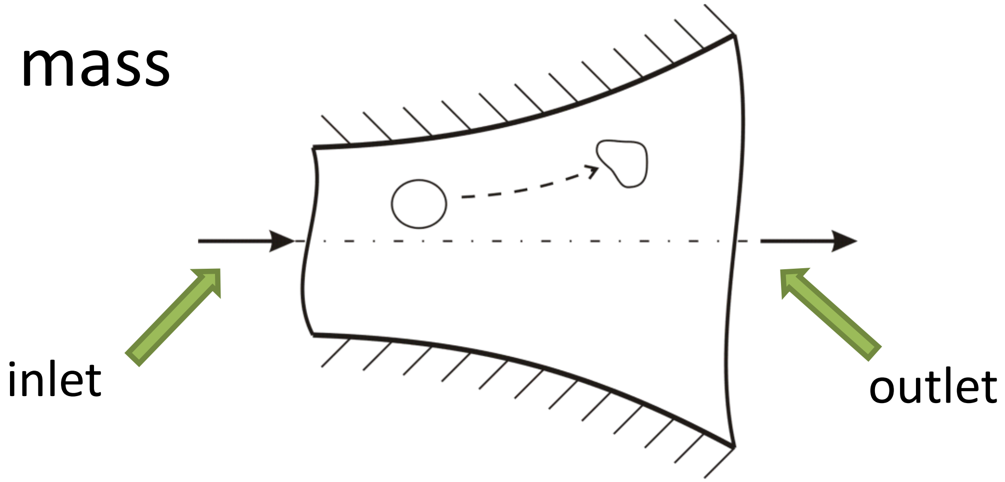


1. Fluid Flow and Thermodynamics

Introduction

- System or Control Mass (CM):
  - is a collection of matter of fixed identity
  - it may be considered enclosed by an invisible, massless, flexible surface through which no matter can pass
  - the boundary of the system may change position, size, and shape
  - is also called control mass

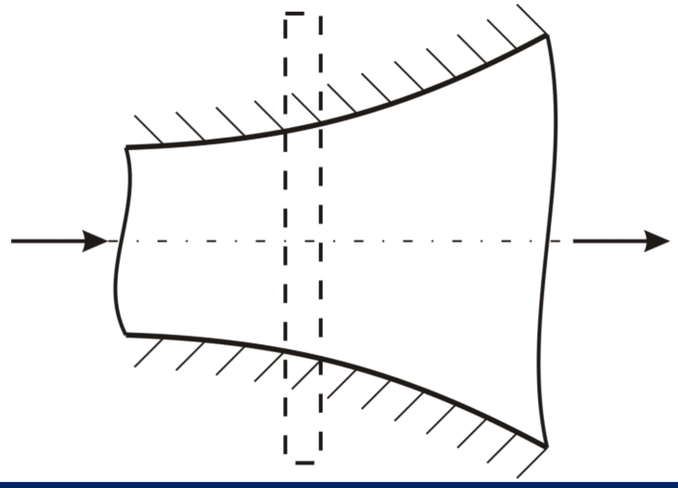
moving of control mass



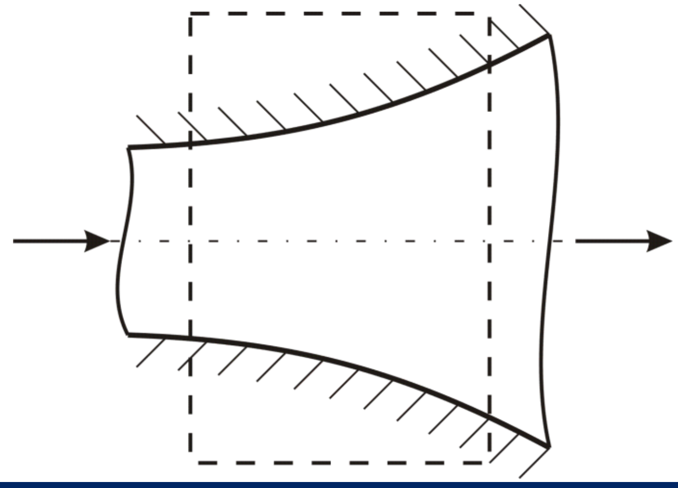
1. Fluid Flow and Thermodynamics

- Control Volume (CV):
  - is arbitrary volume fixed to the coordinate system (stationary or moving)
  - bounded by control surface (CS) through which fluid may pass, CV can has differential or finite size

differential CV



finite CV



1. Fluid Flow and Thermodynamics

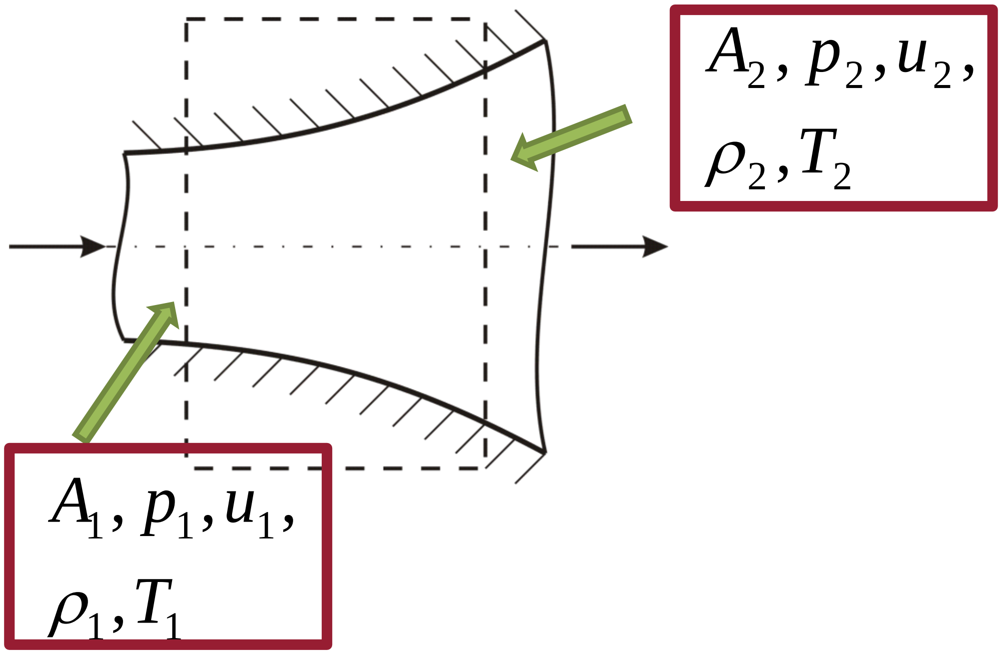
Fundamental equations

- There are 4 fundamental equations, which must be considered:
  - Continuity equation
  - Momentum equation
  - Energy equation
  - Entropy equation

1. Fluid Flow and Thermodynamics

Fundamental equations

- Continuity equation



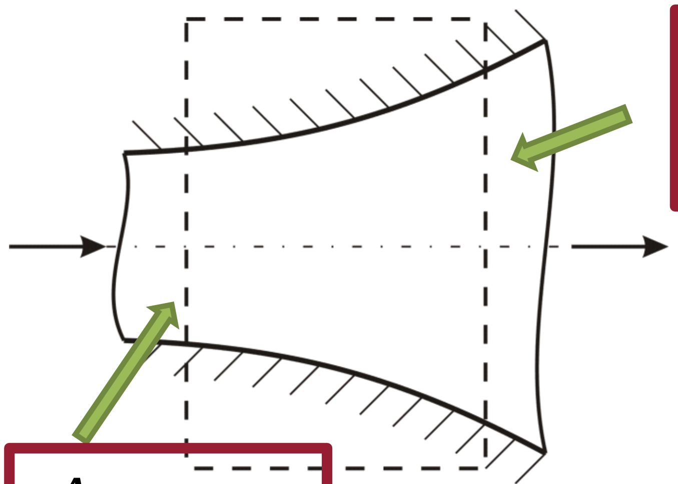
inviscid steady-state flow

$$\begin{bmatrix}\text{Rate mass}\\ \text{leaves CV}\end{bmatrix} - \begin{bmatrix}\text{Rate mass}\\ \text{enters CV}\end{bmatrix} = 0$$

1. Fluid Flow and Thermodynamics

Fundamental equations

- Continuity equation



$A_2, p_2, u_2, \rho_2, T_2$

$A_1, p_1, u_1, \rho_1, T_1$

inviscid steady-state flow

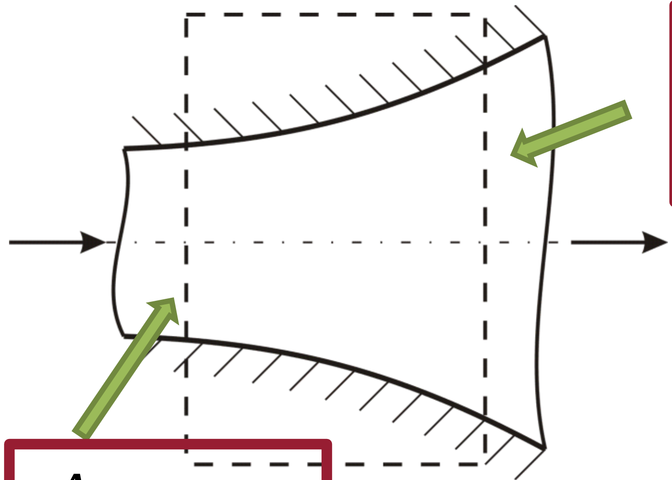
$$\begin{bmatrix} \text{Rate mass} \\ \text{leaves CV} \end{bmatrix} - \begin{bmatrix} \text{Rate mass} \\ \text{enters CV} \end{bmatrix} = 0$$

$$A_2 \rho_2 u_2 \qquad A_1 \rho_1 u_1$$

1. Fluid Flow and Thermodynamics

Fundamental equations

- Continuity equation



$A_2, p_2, u_2, \rho_2, T_2$

$A_1, p_1, u_1, \rho_1, T_1$

inviscid steady-state flow

$$\begin{bmatrix}\text{Rate mass} \\ \text{leaves CV}\end{bmatrix} - \begin{bmatrix}\text{Rate mass} \\ \text{enters CV}\end{bmatrix} = 0$$

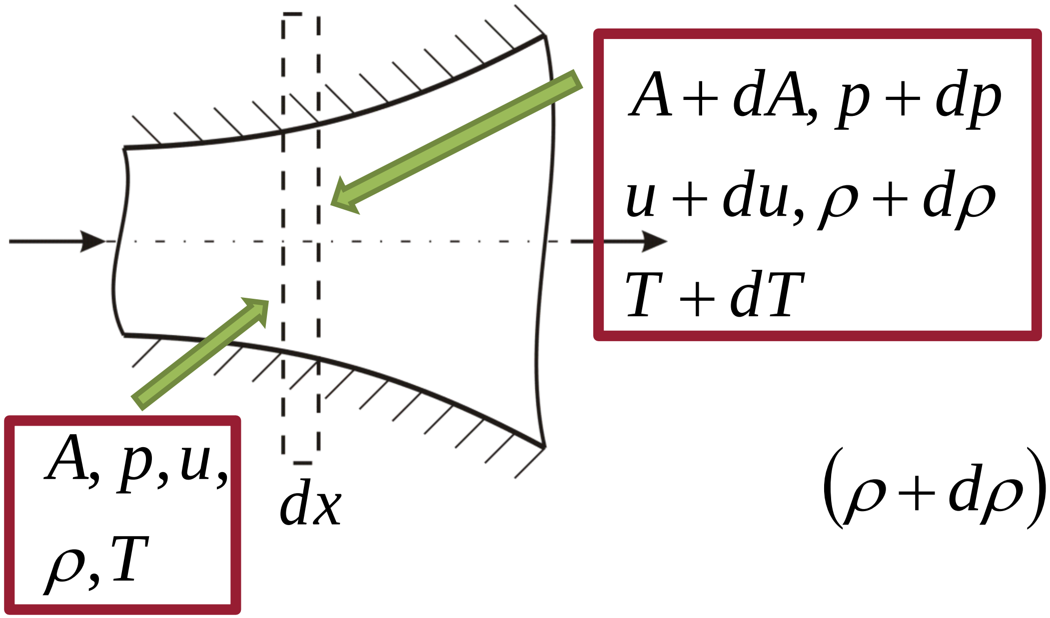
$A_2\rho_2 u_2$ $\quad$ $A_1\rho_1 u_1$

$$A_2\rho_2 u_2 - A_1\rho_1 u_1 = 0$$

1. Fluid Flow and Thermodynamics

Fundamental equations

- Continuity equation



inviscid steady-state flow

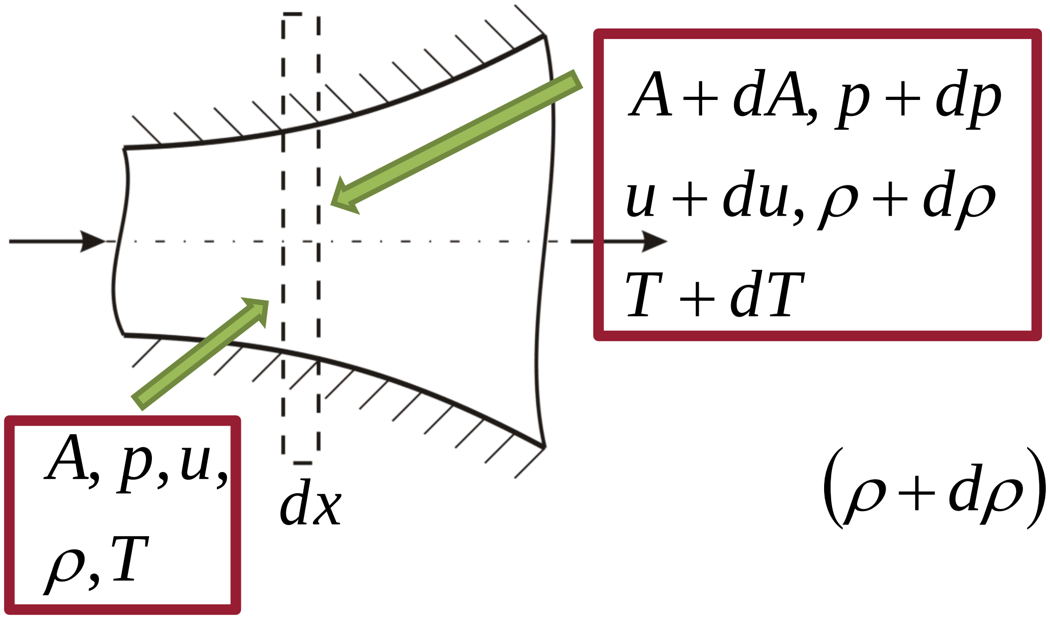
$$\begin{bmatrix}\text{Rate mass}\\ \text{leaves CV}\end{bmatrix} - \begin{bmatrix}\text{Rate mass}\\ \text{enters CV}\end{bmatrix} = 0$$

$$(\rho+d\rho)(u+du)(A+dA) \qquad \rho u A$$

1. Fluid Flow and Thermodynamics

Fundamental equations

- Continuity equation



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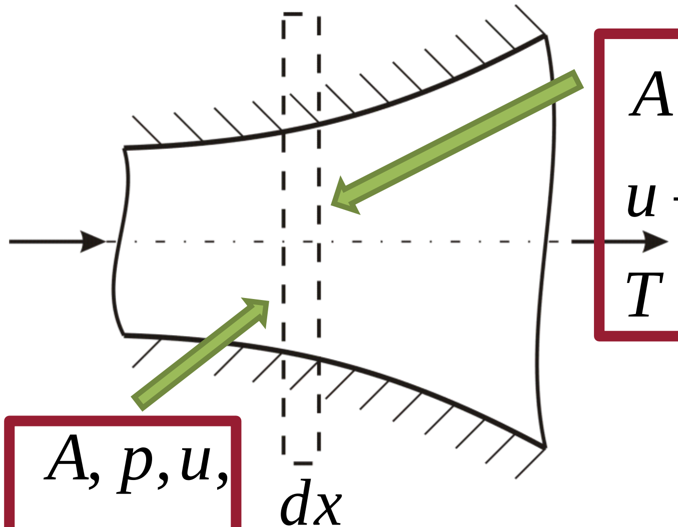
$$\rho u dA + \rho A du + u A d\rho = 0$$

1. Fluid Flow and Thermodynamics

Fundamental equations

- Continuity equation



A, p, u, ρ, T

$A + dA, p + dp$
$u + du, \rho + d\rho$
$T + dT$

inviscid steady-state flow

$$\begin{bmatrix}\text{Rate mass} \\ \text{leaves CV}\end{bmatrix} - \begin{bmatrix}\text{Rate mass} \\ \text{enters CV}\end{bmatrix} = 0$$

$$(\rho + d\rho)(u + du)(A + dA) \qquad \rho u A$$

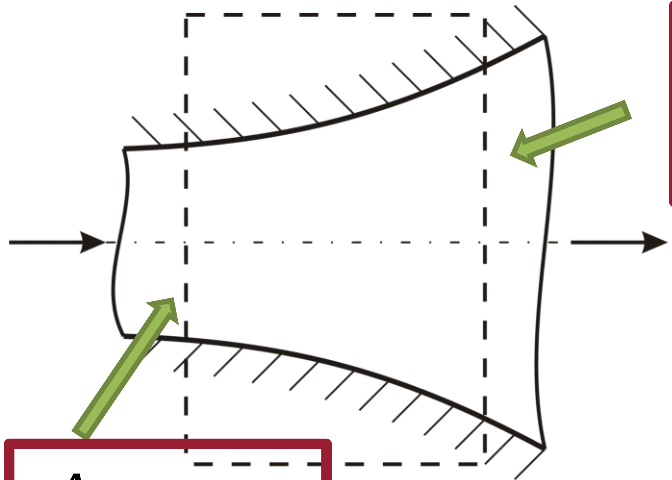
$$\rho u dA + \rho A du + u A d\rho = 0$$

$$\frac{d\rho}{\rho} + \frac{du}{u} + \frac{dA}{A} = 0$$

1. Fluid Flow and Thermodynamics

Fundamental equations

- Momentum equation



$A_2, p_2, u_2, \rho_2, T_2$

$A_1, p_1, u_1, \rho_1, T_1$

inviscid steady-state flow

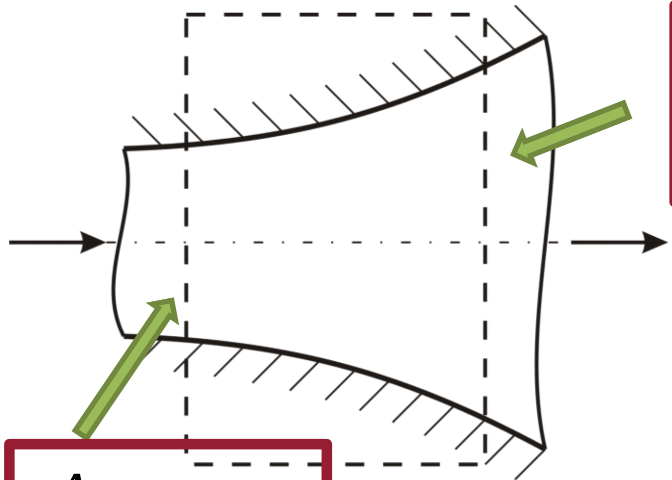
$$\begin{bmatrix} \text{Forces on} \\ \text{gas in CV in} \\ \text{direction } x \end{bmatrix} = \begin{bmatrix} \text{Rate} \\ \text{momentum} \\ \text{leaves CV} \end{bmatrix} - \begin{bmatrix} \text{Rate} \\ \text{momentum} \\ \text{enters CV} \end{bmatrix}$$

1. Fluid Flow and Thermodynamics

Fundamental equations

- Momentum equation

inviscid steady-state flow



$A_2, p_2, u_2, \rho_2, T_2$

$A_1, p_1, u_1, \rho_1, T_1$

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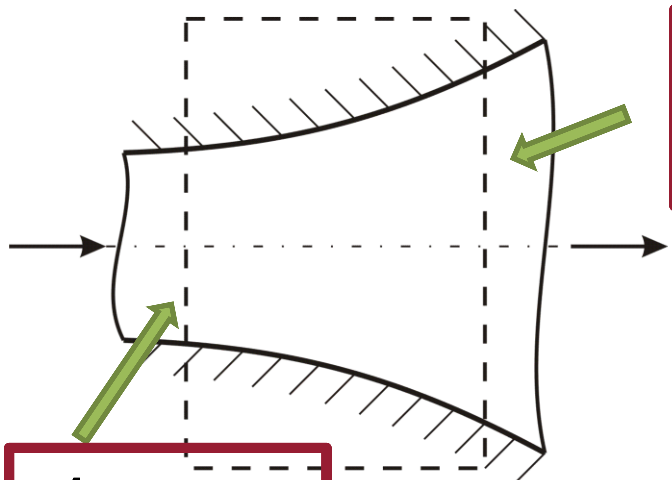
$$\sum F = p_1 A_1 - p_2 A_2 + \int_{A_1}^{A_2} p\,dA \qquad A_2 \rho_2 u_2^2 \qquad A_1 \rho_1 u_1^2$$

1. Fluid Flow and Thermodynamics

Fundamental equations

- Momentum equation



$A_2, p_2, u_2, \rho_2, T_2$

$A_1, p_1, u_1, \rho_1, T_1$

inviscid steady-state flow

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$$\sum F = p_1 A_1 - p_2 A_2 + \int_{A_1}^{A_2} p\,dA \qquad A_2 \rho_2 u_2^2 \qquad A_1 \rho_1 u_1^2$$

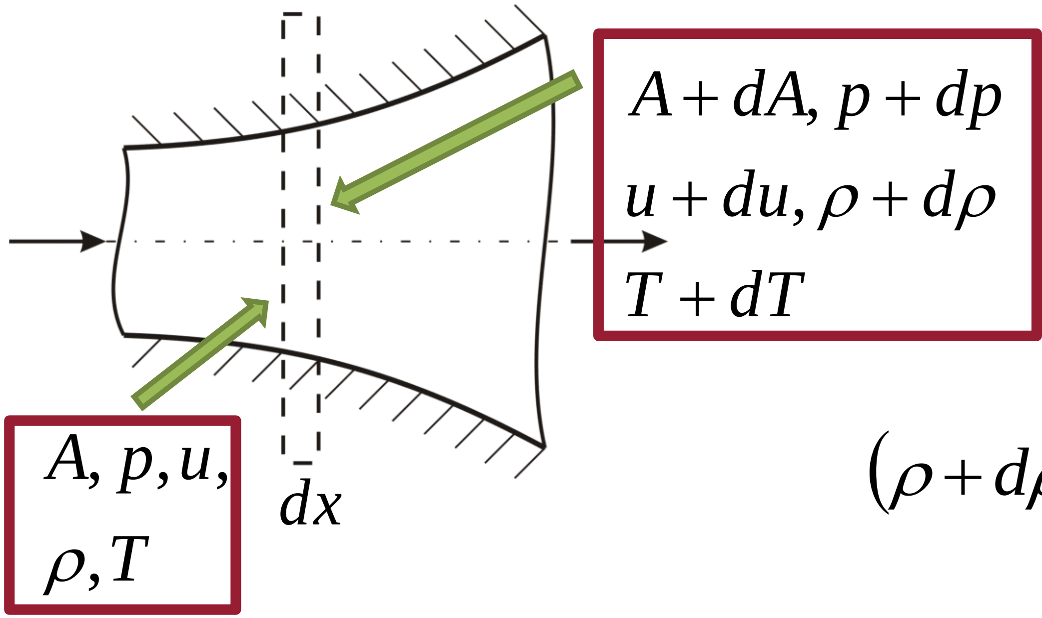
$$p_1 A_1 - p_2 A_2 + \int_{A_1}^{A_2} p\,dA = A_2 \rho_2 u_2^2 - A_1 \rho_1 u_1^2$$

1. Fluid Flow and Thermodynamics

Fundamental equations

- Momentum equation

inviscid steady-state flow



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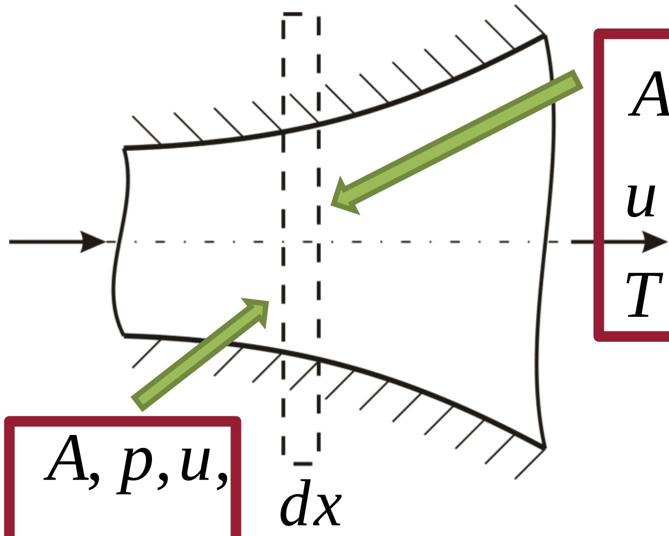
$$(\rho+d\rho)(u+du)^2(A+dA) \qquad \rho u^2 A$$

1. Fluid Flow and Thermodynamics

Fundamental equations

- Momentum equation



$A+dA,\ p+dp$

$u+du,\ \rho+d\rho$

$T+dT$

$A,\ p,\ u,$

$\rho,\ T$

inviscid steady-state flow

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$$(\rho+d\rho)(u+du)^2\,(A+dA) \qquad \rho u^2 A$$

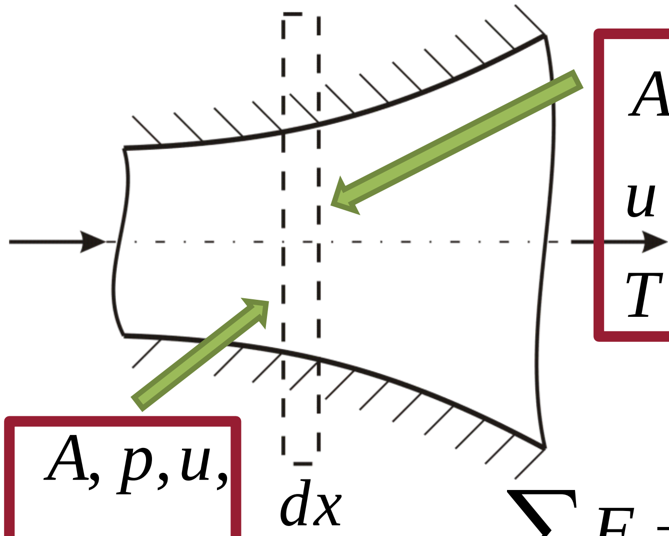
$$A\rho\, u du$$

1. Fluid Flow and Thermodynamics

Fundamental equations

- Momentum equation

inviscid steady-state flow



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$u+du,\ \rho+d\rho$
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$A,\ p,\ u,$
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$$\begin{bmatrix}\text{Forces on} \\ \text{gas in CV in} \\ \text{direction } x\end{bmatrix} = \begin{bmatrix}\text{Rate} \\ \text{momentum} \\ \text{leaves CV}\end{bmatrix} - \begin{bmatrix}\text{Rate} \\ \text{momentum} \\ \text{enters CV}\end{bmatrix}$$

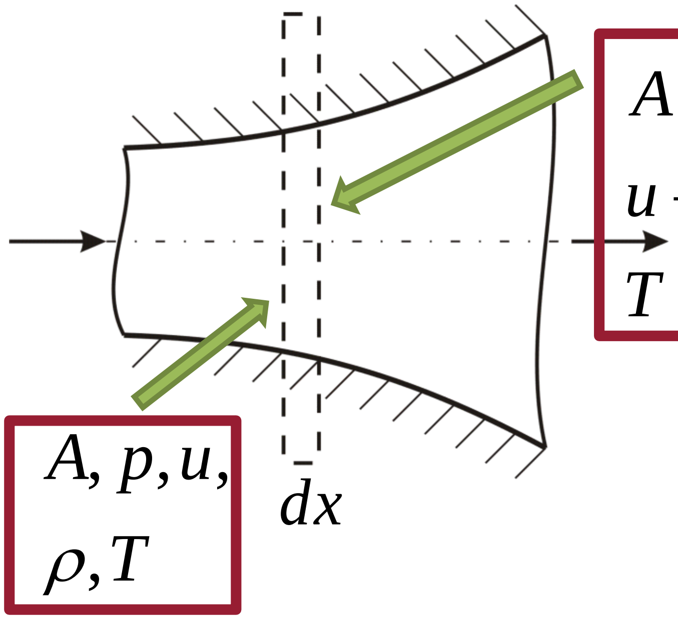
$$\sum F = pA - (p+dp)(A+dA) + $$
$$+ \frac{1}{2}\left(p + (p+dp)\right)\left((A+dA) - A\right) = -A\,dp$$

$$A\rho\, u\, du$$

1. Fluid Flow and Thermodynamics

Fundamental equations

- Momentum equation



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$$-Adp \qquad A\rho\, udu$$

$$-Adp = A\rho\, udu$$

1. Fluid Flow and Thermodynamics

Fundamental equations

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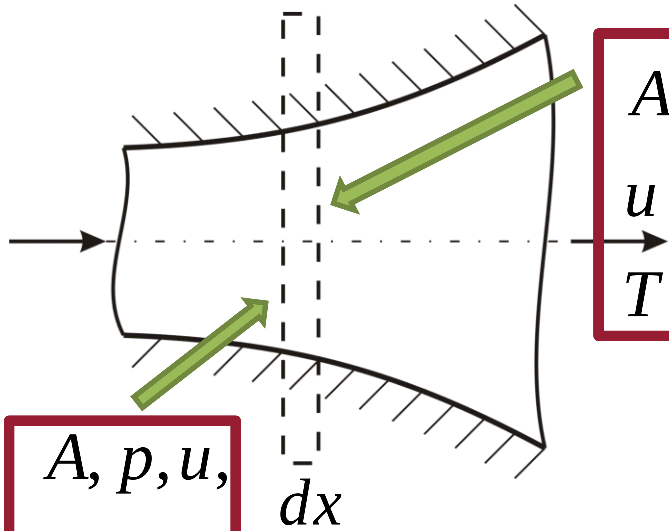
inviscid steady-state flow



$A+dA,\ p+dp$
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$$\begin{bmatrix}\text{Forces on}\\ \text{gas in CV in}\\ \text{direction } x\end{bmatrix}=\begin{bmatrix}\text{Rate}\\ \text{momentum}\\ \text{leaves CV}\end{bmatrix}-\begin{bmatrix}\text{Rate}\\ \text{momentum}\\ \text{enters CV}\end{bmatrix}$$

$$-Adp \qquad A\rho\, udu$$

$$-Adp = A\rho\, udu$$

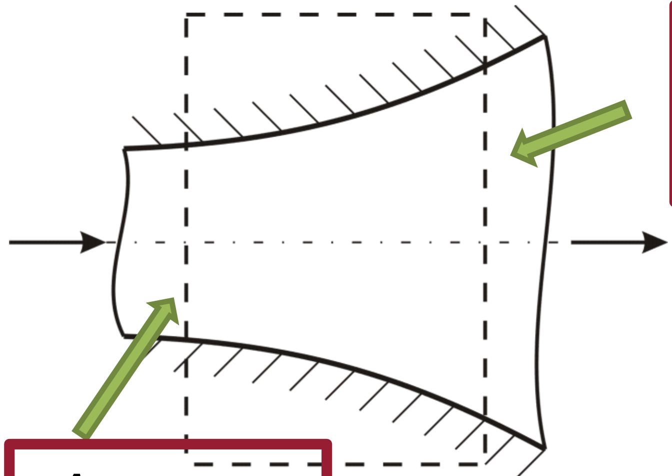
$$-\frac{dp}{\rho}=udu$$

Euler's equation

1. Fluid Flow and Thermodynamics

Fundamental equations

- Energy equation



$A_2, p_2, u_2, \rho_2, T_2$

$A_1, p_1, u_1, \rho_1, T_1$

inviscid steady-state flow

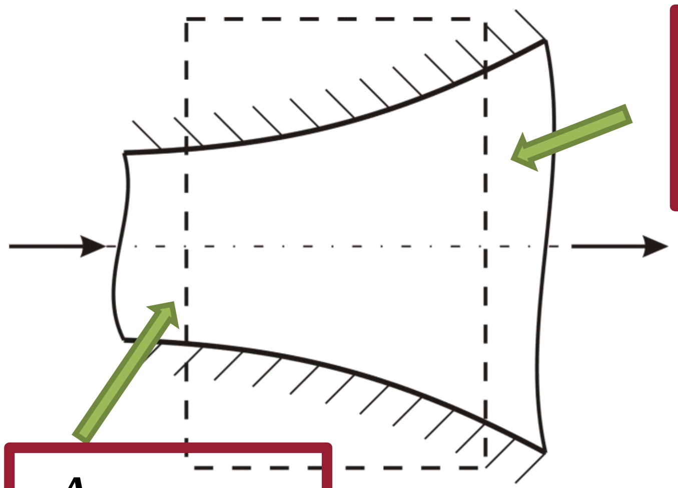
$$\begin{bmatrix}\text{Rate energy} \\ \text{transfer into} \\ \text{CV by work} \\ \text{and heat}\end{bmatrix} = \begin{bmatrix}\text{Rate} \\ \text{energy} \\ \text{leaves CV}\end{bmatrix} - \begin{bmatrix}\text{Rate} \\ \text{energy} \\ \text{enters CV}\end{bmatrix}$$

1. Fluid Flow and Thermodynamics

Fundamental equations

- Energy equation

inviscid steady-state flow



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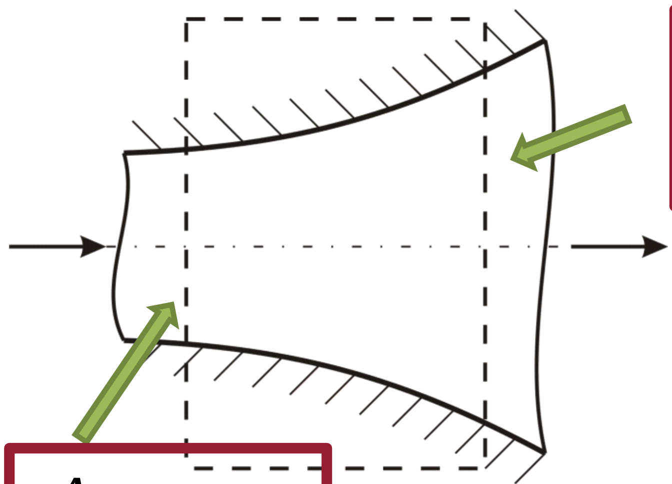
$$\dot{W}_{\text{in}} + \dot{Q}_{\text{in}} = \rho_2 u_2 A_2 \left(e_2 + \frac{u_2^2}{2} \right) - \rho_1 u_1 A_1 \left(e_1 + \frac{u_1^2}{2} \right)$$

1. Fluid Flow and Thermodynamics

Fundamental equations

- Energy equation



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$$\rho_2 u_2 A_2 \left(e_2 + \frac{u_2^2}{2} \right) \qquad \rho_1 u_1 A_1 \left(e_1 + \frac{u_1^2}{2} \right)$$

$$\dot{W}_{\text{in}} + \dot{Q}_{\text{in}}$$

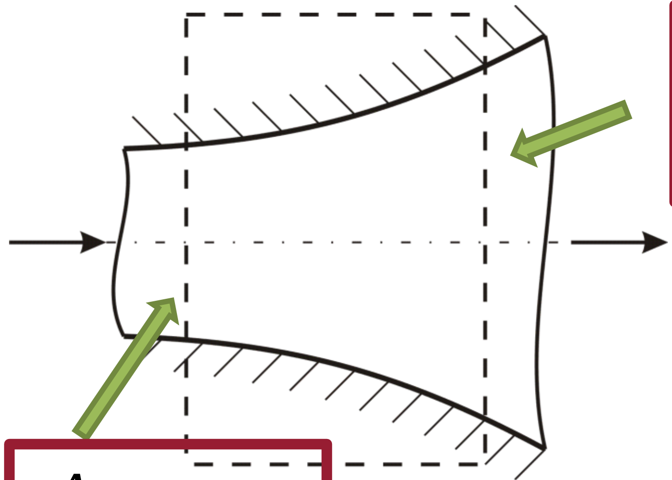
$$\dot{W}_{\text{p,in}} + \dot{W}_{\text{s,in}}$$

1. Fluid Flow and Thermodynamics

Fundamental equations

- Energy equation

inviscid steady-state flow



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$$\rho_2 u_2 A_2\left(e_2 + \frac{u_2^2}{2}\right) \qquad \rho_1 u_1 A_1\left(e_1 + \frac{u_1^2}{2}\right)$$

$$\dot{W}_{\text{in}} + \dot{Q}_{\text{in}}$$

$$\dot{W}_{\text{p,in}} + \dot{W}_{\text{s,in}}$$

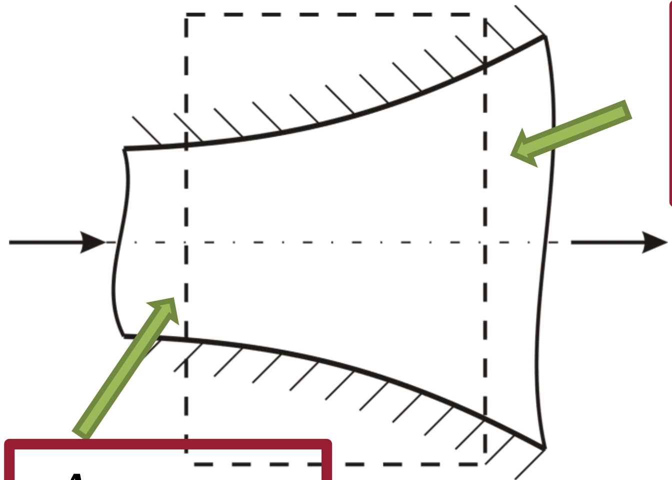
$$\frac{p_1}{\rho_1}\rho_1 u_1 A_1 - \frac{p_2}{\rho_2}\rho_2 u_2 A_2$$

1. Fluid Flow and Thermodynamics

Fundamental equations

- Energy equation

inviscid steady-state flow



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$$\rho_2 u_2 A_2 \left(e_2 + \frac{u_2^2}{2} \right) \qquad \rho_1 u_1 A_1 \left(e_1 + \frac{u_1^2}{2} \right)$$

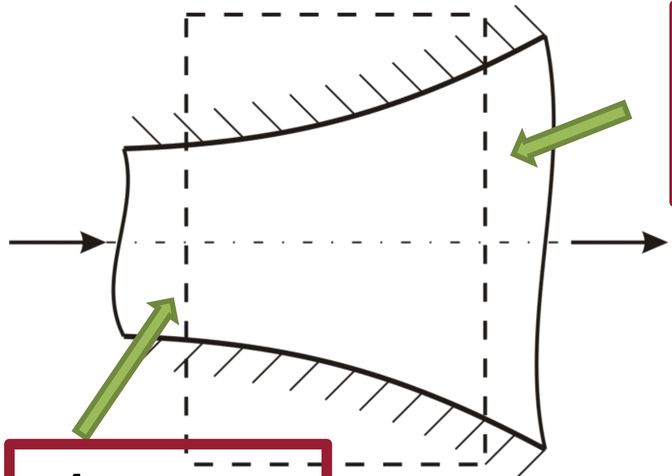
$$\frac{p_1}{\rho_1} \rho_1 u_1 A_1 - \frac{p_2}{\rho_2} \rho_2 u_2 A_2 + \dot{W}_{\text{s,in}} + \dot{Q}_{\text{in}}$$

$$\dot{W}_{\text{s,in}} + \dot{Q}_{\text{in}} = \rho_2 u_2 A_2 \left(e_2 + \frac{p_2}{\rho_2} + \frac{u_2^2}{2} \right) - \rho_1 u_1 A_1 \left(e_1 + \frac{p_1}{\rho_1} + \frac{u_1^2}{2} \right)$$

1. Fluid Flow and Thermodynamics

Fundamental equations

- Energy equation



$A_2, p_2, u_2, \rho_2, T_2$

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inviscid steady-state flow

$$\begin{bmatrix}\text{Rate energy}\\ \text{transfer into}\\ \text{CV by work}\\ \text{and heat}\end{bmatrix} = \begin{bmatrix}\text{Rate}\\ \text{energy}\\ \text{leaves CV}\end{bmatrix} - \begin{bmatrix}\text{Rate}\\ \text{energy}\\ \text{enters CV}\end{bmatrix}$$

using enthalpy *h*

$$\dot{W}_{s,in} + \dot{Q}_{in} = \rho_2 u_2 A_2 \left(h_2 + \frac{u_2^2}{2} \right) - \rho_1 u_1 A_1 \left(h_1 + \frac{u_1^2}{2} \right)$$

1. Fluid Flow and Thermodynamics

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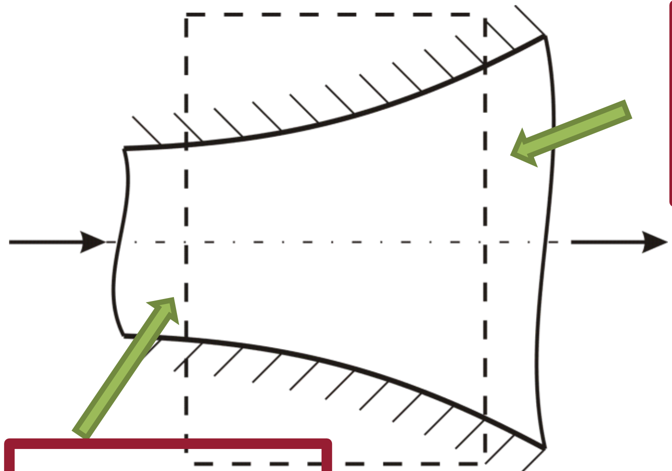

$A_2, p_2, u_2,$
ρ_2, T_2

$A_1, p_1, u_1,$
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using enthalpy *h*

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adiabatic process
no shaft work

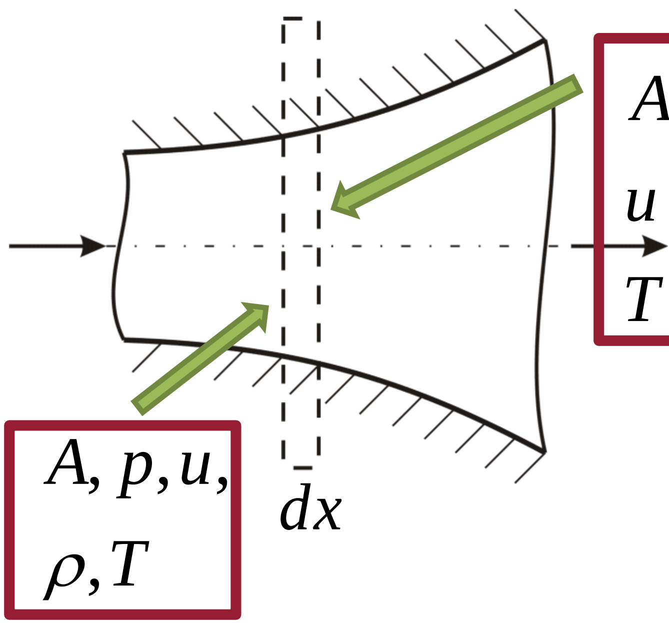
$$\left(h_1 + \frac{u_1^2}{2} \right) = \left(h_2 + \frac{u_2^2}{2} \right)$$

1. Fluid Flow and Thermodynamics

Fundamental equations

- Energy equation

inviscid steady-state flow



$A + dA,\ p + dp$

$u + du,\ \rho + d\rho$

$T + dT$

$$\begin{bmatrix}\text{Rate energy}\\ \text{transfer into}\\ \text{CV by work}\\ \text{and heat}\end{bmatrix} = \begin{bmatrix}\text{Rate}\\ \text{energy}\\ \text{leaves CV}\end{bmatrix} - \begin{bmatrix}\text{Rate}\\ \text{energy}\\ \text{enters CV}\end{bmatrix}$$

$$+(\rho + d\rho)(u + du)(A + dA)\cdot$$
$$\cdot\left(e + de + (u + du)^2/2\right)$$

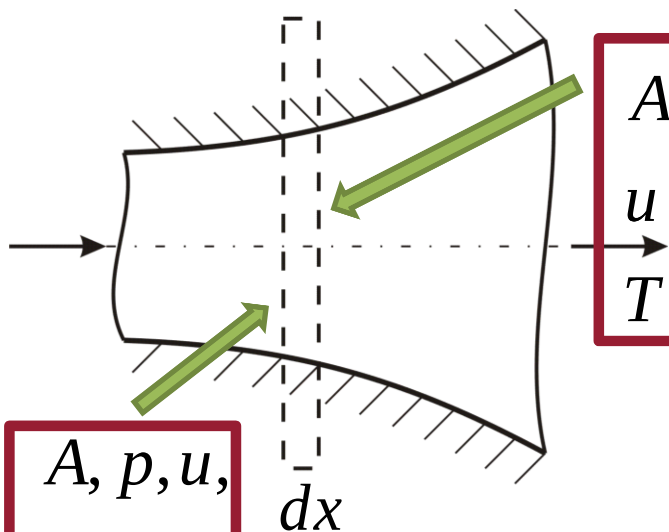
$$\rho u A\left(e + u^2/2\right)$$

1. Fluid Flow and Thermodynamics

Fundamental equations

- Energy equation

inviscid steady-state flow



A, p, u, ρ, T

$A+dA,\ p+dp,\ u+du,\ \rho+d\rho,\ T+dT$

$$\begin{bmatrix}\text{Rate energy}\\ \text{transfer into}\\ \text{CV by work}\\ \text{and heat}\end{bmatrix} = \begin{bmatrix}\text{Rate}\\ \text{energy}\\ \text{leaves CV}\end{bmatrix} - \begin{bmatrix}\text{Rate}\\ \text{energy}\\ \text{enters CV}\end{bmatrix}$$

$$+(\rho+d\rho)(u+du)(A+dA)\cdot\left(e+de+(u+du)^2/2\right) \qquad \rho u A\left(e+u^2/2\right)$$

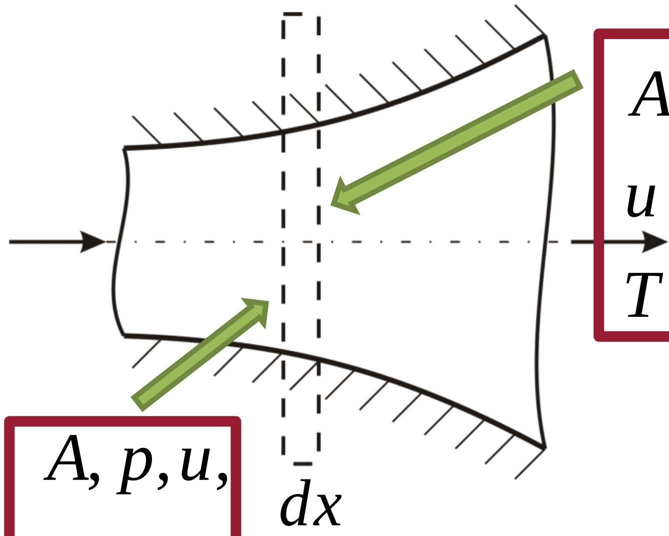
$$\rho u A(de+udu)$$

1. Fluid Flow and Thermodynamics

Fundamental equations

- Energy equation

inviscid steady-state flow



$A+dA,\ p+dp$
$u+du,\ \rho+d\rho$
$T+dT$

$A,\ p, u,$
ρ, T

$$\begin{bmatrix}\text{Rate energy} \\ \text{transfer into} \\ \text{CV by work} \\ \text{and heat}\end{bmatrix} = \begin{bmatrix}\text{Rate} \\ \text{energy} \\ \text{leaves CV}\end{bmatrix} - \begin{bmatrix}\text{Rate} \\ \text{energy} \\ \text{enters CV}\end{bmatrix}$$

$$\delta\dot{W}_{\text{in}} + \delta\dot{Q}_{\text{in}}$$

$$\rho u A(de + udu)$$

$$\delta\dot{W}_{\text{p,in}} + \delta\dot{W}_{\text{s,in}}$$

$$p\,uA - (p+dp)(u+du)(A+dA) \Rightarrow -pAdu - Audp - pudA$$

1. Fluid Flow and Thermodynamics

Fundamental equations

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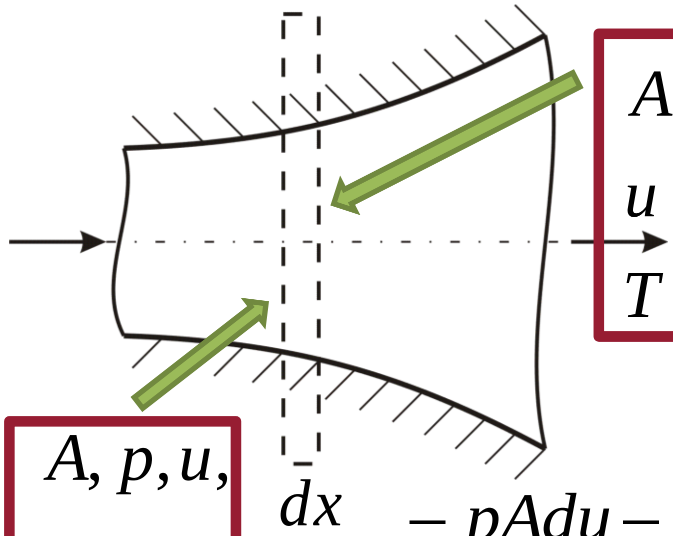

$$A,\ p,\ u,\ \rho,\ T$$

$$A+dA,\ p+dp,\ u+du,\ \rho+d\rho,\ T+dT$$

$$\begin{bmatrix}\text{Rate energy}\\ \text{transfer into}\\ \text{CV by work}\\ \text{and heat}\end{bmatrix} = \begin{bmatrix}\text{Rate}\\ \text{energy}\\ \text{leaves CV}\end{bmatrix} - \begin{bmatrix}\text{Rate}\\ \text{energy}\\ \text{enters CV}\end{bmatrix}$$

$$-pAdu - Audp - pudA + \delta\dot{W}_{\mathrm{s,in}} + \delta\dot{Q}_{\mathrm{in}}$$

$$\rho u A\left(de + udu\right)$$

$$-d\left(p/\rho\right) + \delta w_{\mathrm{s,in}} + \delta q_{\mathrm{in}} = $$
$$= de + udu$$

1. Fluid Flow and Thermodynamics

Fundamental equations

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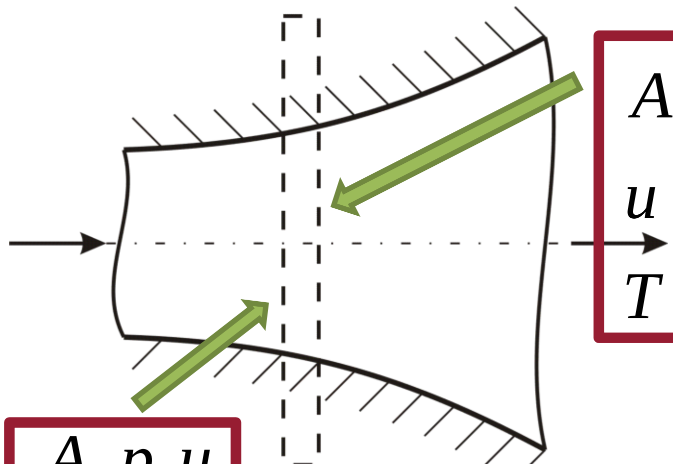
inviscid steady-state flow



A, p, u, ρ, T

dx

$A+dA,\ p+dp$
$u+du,\ \rho+d\rho$
$T+dT$

$$\begin{bmatrix}\text{Rate energy} \\ \text{transfer into} \\ \text{CV by work} \\ \text{and heat}\end{bmatrix} = \begin{bmatrix}\text{Rate} \\ \text{energy} \\ \text{leaves CV}\end{bmatrix} - \begin{bmatrix}\text{Rate} \\ \text{energy} \\ \text{enters CV}\end{bmatrix}$$

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$$\rho u A(de + udu)$$

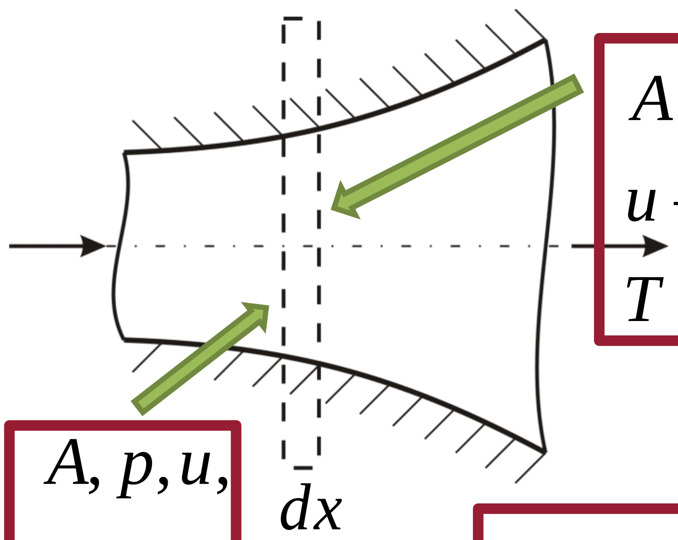
$$-d(p/\rho) + \delta w_{\mathrm{s,in}} + \delta q_{\mathrm{in}} = $$
$$= de + udu$$

using enthalpy *h*

$$\delta w_{\mathrm{s,in}} + \delta q_{\mathrm{in}} = dh + udu$$

Fundamental equations

- Energy equation



A, p, u, ρ, T

$A+dA, p+dp$
$u+du, \rho+d\rho$
$T+dT$

inviscid steady-state flow

$$\begin{bmatrix}\text{Rate energy} \\ \text{transfer into} \\ \text{CV by work} \\ \text{and heat}\end{bmatrix} = \begin{bmatrix}\text{Rate} \\ \text{energy} \\ \text{leaves CV}\end{bmatrix} - \begin{bmatrix}\text{Rate} \\ \text{energy} \\ \text{enters CV}\end{bmatrix}$$

$$\delta w_{\text{s,in}} + \delta q_{\text{in}} = dh + u\,du$$

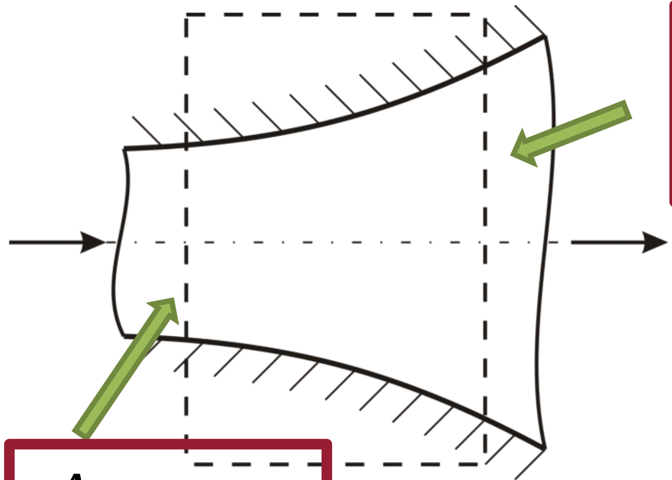
adiabatic process
no shaft work

$$dh + u\,du = 0$$

1. Fluid Flow and Thermodynamics

Fundamental equations

- Entropy equation



$A_2, p_2, u_2, \rho_2, T_2$

$A_1, p_1, u_1, \rho_1, T_1$

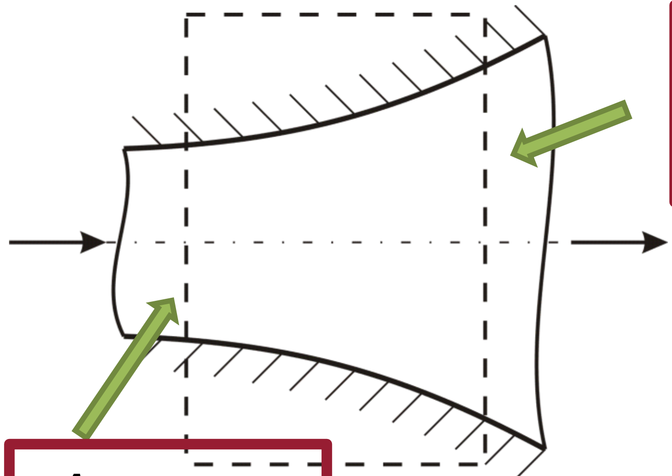
inviscid steady-state flow

$$\begin{bmatrix} \text{Rate entropy} \\ \text{transfer into} \\ \text{CV by heat and} \\ \text{irreversibilities} \end{bmatrix} = \begin{bmatrix} \text{Rate} \\ \text{entropy} \\ \text{leaves} \\ \text{CV} \end{bmatrix} - \begin{bmatrix} \text{Rate} \\ \text{entropy} \\ \text{enters} \\ \text{CV} \end{bmatrix}$$

1. Fluid Flow and Thermodynamics

Fundamental equations

- Entropy equation



inviscid steady-state flow

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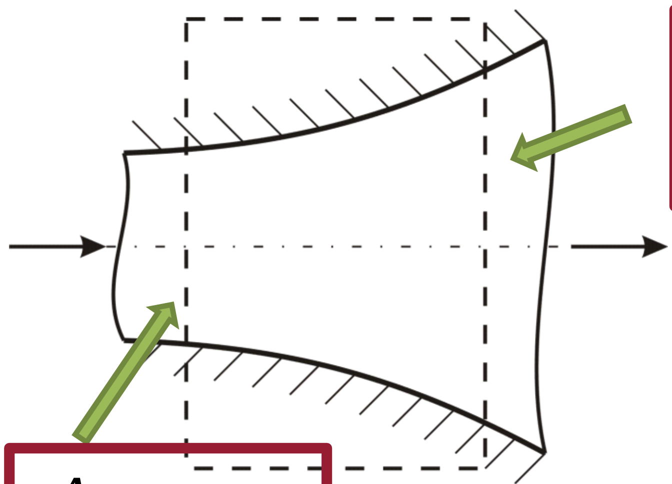
$$\int_1^2 \frac{\delta \dot{Q}_{\text{in}}}{T} + \dot{S}_{\text{gen}} \qquad \dot{S}_2 \qquad \dot{S}_1$$

1. Fluid Flow and Thermodynamics

Fundamental equations

- Entropy equation



$A_2, p_2, u_2, \rho_2, T_2$

$A_1, p_1, u_1, \rho_1, T_1$

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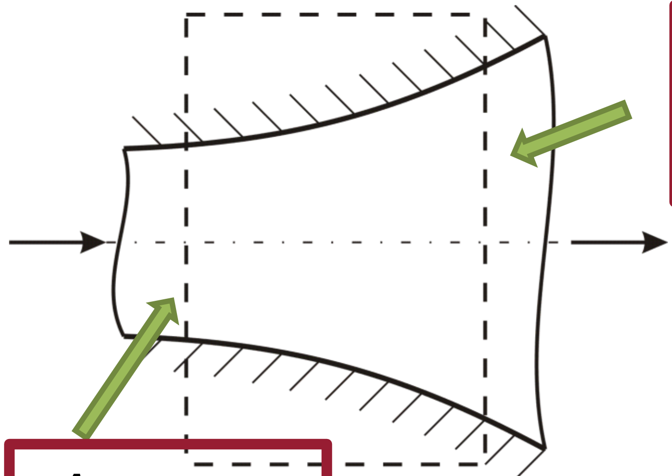
1. Fluid Flow and Thermodynamics

Fundamental equations

- Entropy equation



$A_1, p_1, u_1, \rho_1, T_1$

$A_2, p_2, u_2, \rho_2, T_2$

inviscid steady-state flow

$$\begin{bmatrix}\text{Rate entropy} \\ \text{transfer into} \\ \text{CV by heat and} \\ \text{irreversibilities}\end{bmatrix} = \begin{bmatrix}\text{Rate} \\ \text{entropy} \\ \text{leaves} \\ \text{CV}\end{bmatrix} - \begin{bmatrix}\text{Rate} \\ \text{entropy} \\ \text{enters} \\ \text{CV}\end{bmatrix}$$

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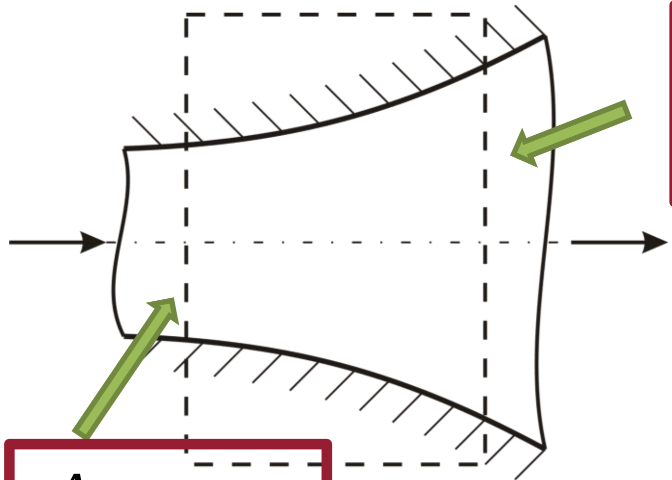
reversible process 1-2

$$\int_1^2 \frac{\delta \dot{Q}_{in}}{T} = \dot{S}_2 - \dot{S}_1$$

1. Fluid Flow and Thermodynamics

Fundamental equations

- Entropy equation



$A_2, p_2, u_2, \rho_2, T_2$

$A_1, p_1, u_1, \rho_1, T_1$

inviscid steady-state flow

$$\begin{bmatrix}\text{Rate entropy transfer into CV by heat and irreversibilities}\end{bmatrix} = \begin{bmatrix}\text{Rate entropy leaves CV}\end{bmatrix} - \begin{bmatrix}\text{Rate entropy enters CV}\end{bmatrix}$$

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adiabatic reversible process 1-2

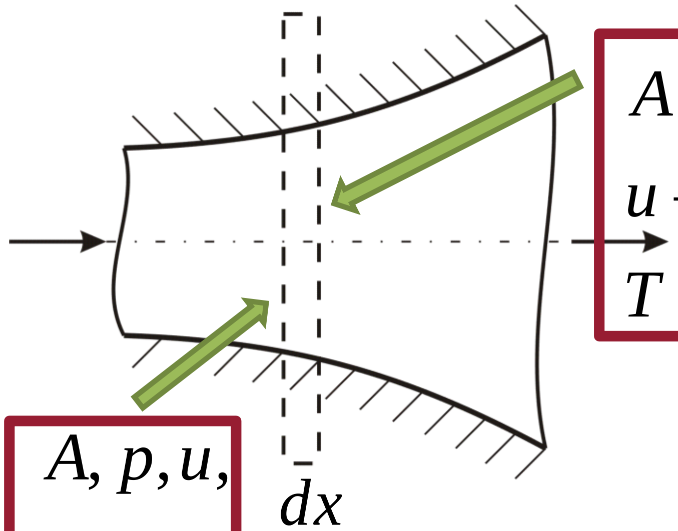
isentropic process 1-2

$$0 = \dot{S}_2 - \dot{S}_1$$

1. Fluid Flow and Thermodynamics

Fundamental equations

- Entropy equation



A, p, u, ρ, T

$A + dA, p + dp$
$u + du, \rho + d\rho$
$T + dT$

inviscid steady-state flow

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$$\frac{\delta \dot{Q}_{\text{in}}}{T} + d\dot{S}_{\text{gen}} \qquad \dot{S} + d\dot{S} \qquad \dot{S}$$

$$\frac{\delta \dot{Q}_{\text{in}}}{T} + d\dot{S}_{\text{gen}} = d\dot{S}$$

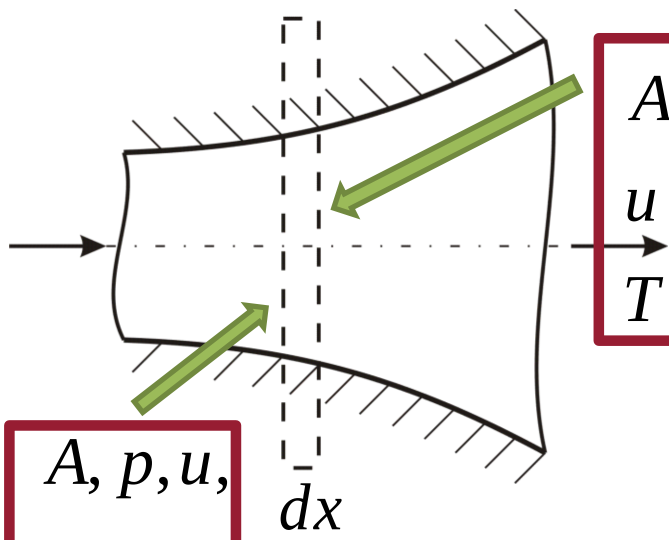
reversible process

$$\frac{\delta \dot{Q}_{\text{in}}}{T} = d\dot{S}, \quad \frac{\delta \dot{q}_{\text{in}}}{T} = d\dot{s}$$

1. Fluid Flow and Thermodynamics

Fundamental equations

- Entropy equation



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$$\frac{\delta \dot{Q}_{\text{in}}}{T} + d\dot{S}_{\text{gen}} = d\dot{S}$$

adiabatic reversible process

isentropic process

$$0 = d\dot{S}, 0 = d\dot{s}, 0 = ds$$

1. Fluid Flow and Thermodynamics

Fundamental equations

inviscid steady-state flow

Quasi-one dimension fluid flow equations:

- Continuity equation → $$\frac{d\rho}{\rho}+\frac{du}{u}+\frac{dA}{A}=0$$
- Momentum equation → $$-\frac{dp}{\rho}=udu$$
- Energy equation → $$dh+udu=0$$ adiabatic flow, no shaft work
- Entropy equation → $$0=ds$$ isentropic flow

1. Fluid Flow and Thermodynamics

Thermodynamics of gases

- perfect gas
  - equation of state (EOS) $p = \rho RT$, $pv = RT$ or $p = \rho R_M T / M_m$, $pv = R_M T / M_m$

1. Fluid Flow and Thermodynamics

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- perfect gas
  - equation of state (EOS) $p = \rho RT$ or $p = \rho R_M T / M_m$

    $pv = RT$ $\quad pv = R_M T / M_m$

  - internal energy e → $e = e(T)$ → $de = c_v dT$
  - enthalpy $h = e + pv$ → $h = h(T)$ → $dh = c_p dT$

1. Fluid Flow and Thermodynamics

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  - internal energy e
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$$e = e(T) \qquad h = h(T)$$

$$de = c_v dT \qquad dh = c_p dT$$

calorically perfect gas:

$$e = c_v T \qquad h = c_p T$$

1. Fluid Flow and Thermodynamics

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  - internal energy e
  - enthalpy $h = e + pv$

$$e = e(T) \quad h = h(T) \quad \Rightarrow \quad de = c_v dT \quad dh = c_p dT$$

$$e = c_v T \quad h = c_p T$$

calorically perfect gas

  - heat capacity ratio $\gamma = c_p / c_v$
  - difference between heat capacities $R = c_p - c_v$

1. Fluid Flow and Thermodynamics

Thermodynamics of gases

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    $pv = RT$ $\quad pv = R_M T / M_m$
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  - enthalpy $h = e + pv$

$$e = e(T) \qquad de = c_v dT$$
$$h = h(T) \qquad dh = c_p dT$$
$$e = c_v T$$
$$h = c_p T$$

calorically perfect gas

  - heat capacity ratio $\gamma = c_p / c_v$
  - difference between heat capacities $R = c_p - c_v$

$$c_p = \frac{\gamma R}{\gamma - 1} \qquad c_v = \frac{R}{\gamma - 1}$$

1. Fluid Flow and Thermodynamics

Thermodynamics of gases

- moving fluid with speed u
  - static parameters ⇒ p, T, ρ ⇒ “measured“ in moving fluid

1. Fluid Flow and Thermodynamics

Thermodynamics of gases

- moving fluid with speed u
  - static parameters → p, T, ρ → "measured" in moving fluid

$u_0 = 0$

- fluid brought to rest adiabatically → p_0, T_0, ρ_0

1. Fluid Flow and Thermodynamics

Thermodynamics of gases

- moving fluid with speed u
  - static parameters → p, T, ρ → "measured" in moving fluid
    ↓ $u_0 = 0$
  - fluid brought to rest adiabatically → p_0, T_0, ρ_0
    ↓
    total (stagnation) pressure, temperature, density
  - corresponding total enthalpy to total temperature → $h_0 = c_p T_0$

1. Fluid Flow and Thermodynamics

Thermodynamics of gases

- stagnation conditions
  - adiabatic process

total enthalpy is constant through steady, inviscid, adiabatic flow

$$\left(h_1 + \frac{u_1^2}{2}\right) = \text{const.}$$

rest of fluid

$$h_0 = \left(h_1 + \frac{u_1^2}{2}\right)$$

1. Fluid Flow and Thermodynamics

Thermodynamics of gases

- stagnation conditions
  - adiabatic process

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$$h_0 = \left(h_1 + \frac{u_1^2}{2} \right)$$

total enthalpy is constant through steady, inviscid, adiabatic flow

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also **total temperature** is constant

1. Fluid Flow and Thermodynamics

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total enthalpy is constant through steady, inviscid, adiabatic flow

$$h_0 = c_p T_0$$

also **total temperature** is constant

  - isentropic process

$$p_0, \rho_0$$

total pressure and **total density** are constant

1. Fluid Flow and Thermodynamics

- moving fluid with speed u
  - static parameters ⇨ p, T, ρ ⇨ "measured" in moving fluid

1. Fluid Flow and Thermodynamics

Thermodynamics of gases

- moving fluid with speed u
  - static parameters → p, T, ρ → "measured" in moving fluid

  ↓ $u = a^*$

  - fluid brought to speed of sound isentropically → p^*, T^*, ρ^*

1. Fluid Flow and Thermodynamics

Thermodynamics of gases

- moving fluid with speed u
  - static parameters → p, T, ρ → "measured" in moving fluid

    ↓ $u = a^*$

  - fluid brought to speed of sound isentropically → p^*, T^*, ρ^*

    ↓

critical pressure, temperature, density

1. Fluid Flow and Thermodynamics

Thermodynamics of gases

- reversible process
  - added heat – closed system → $\delta w_{\text{in}} + \delta q_{\text{in}} = de$
  - reversible process → $\delta q_{\text{in}} = ds T$
  - internal energy → $de = c_v dT$
  - pressure work → $\delta w_{\text{in}} = -p dv$

1. Fluid Flow and Thermodynamics

Thermodynamics of gases

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  - reversible process → $\delta q_{\text{in}} = dsT$
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using c_v

$$-pdv + dsT = c_v dT$$

1. Fluid Flow and Thermodynamics

- reversible process
  - added heat – closed system → $\delta w_{\text{in}} + \delta q_{\text{in}} = de$
  - reversible process → $\delta q_{\text{in}} = dsT$
  - internal energy → $de = c_v dT$
  - pressure work → $\delta w_{\text{in}} = -pdv$

using c_v:

$$-pdv + dsT = c_v dT$$

using c_p:

$$-pdv + d(pv) + dsT = de + d(pv)$$

$$dpv + dsT = c_p dT$$

1. Fluid Flow and Thermodynamics

Thermodynamics of gases

- reversible process
  - added heat – closed system → $\delta w_{\text{in}} + \delta q_{\text{in}} = de$
  - reversible process → $\delta q_{\text{in}} = dsT$
  - internal energy → $de = c_v dT$
  - pressure work → $\delta w_{\text{in}} = -pdv$

using c_v:

$$-pdv + dsT = c_v dT$$

using c_p:

$$-pdv + d(pv) + dsT = de + d(pv)$$

$$dpv + dsT = c_p dT$$

$$ds = c_v \frac{dT}{T} + \frac{pdv}{T}$$

$$ds = c_p \frac{dT}{T} - \frac{vdp}{T}$$

1. Fluid Flow and Thermodynamics

Thermodynamics of gases

- reversible process
  - added heat – closed system ➡ $\delta w_{\text{in}} + \delta q_{\text{in}} = de$
  - reversible process ➡ $\delta q_{\text{in}} = dsT$
  - internal energy ➡ $de = c_v dT$
  - pressure work ➡ $\delta w_{\text{in}} = -pdv$

$$ds = c_v \frac{dT}{T} + \frac{pdv}{T}$$

$$ds = c_p \frac{dT}{T} - \frac{vdp}{T}$$

➡ isentropic process

$$0 = c_v \frac{dT}{T} + \frac{pdv}{T}$$

$$0 = c_p \frac{dT}{T} - \frac{vdp}{T}$$

1. Fluid Flow and Thermodynamics

Thermodynamics of gases

- reversible process
  - added heat – closed system → $\delta w_{\text{in}} + \delta q_{\text{in}} = de$
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$$0 = c_v \frac{dT}{T} + \frac{pdv}{T}$$

$$0 = c_p \frac{dT}{T} - \frac{vdp}{T}$$

using EOS ↓

$$0 = c_v \frac{dT}{T} + \frac{Rdv}{v}$$

$$0 = c_p \frac{dT}{T} - \frac{Rdp}{p}$$

$$\gamma c_v = c_p = \frac{\gamma R}{\gamma - 1}$$

$$\frac{v_2}{v_1} = \left(\frac{T_2}{T_1}\right)^{\frac{-1}{\gamma-1}}$$

$$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$$

1. Fluid Flow and Thermodynamics

Thermodynamics of gases

- reversible process
  - added heat – closed system → $\delta w_{\text{in}} + \delta q_{\text{in}} = de$
  - reversible process → $\delta q_{\text{in}} = ds T$
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$$\frac{v_2}{v_1} = \left(\frac{T_2}{T_1}\right)^{\frac{-1}{\gamma-1}}$$

$$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{v_1}{v_2}\right)^{\gamma} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma}$$

isentropic process

1. Fluid Flow and Thermodynamics

Speed of sound

- physical mechanism:
  - sound propagation in gas is based on molecular motion
  - energy is transfer to gas molecules, they start to move about in random fashion
  - they collide with other molecules and transfer their energy to these molecules
  - the process of collision repeats – energy is propagated
  - macroscopic parameters p, T, ρ are slightly varied by increased microscopic parameter – energy of molecule

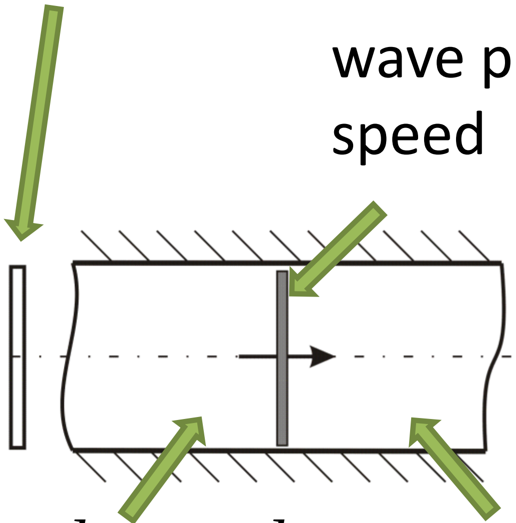
1. Fluid Flow and Thermodynamics

Speed of sound

- parameters of fluid

source of disturbance

wave propagates with speed a



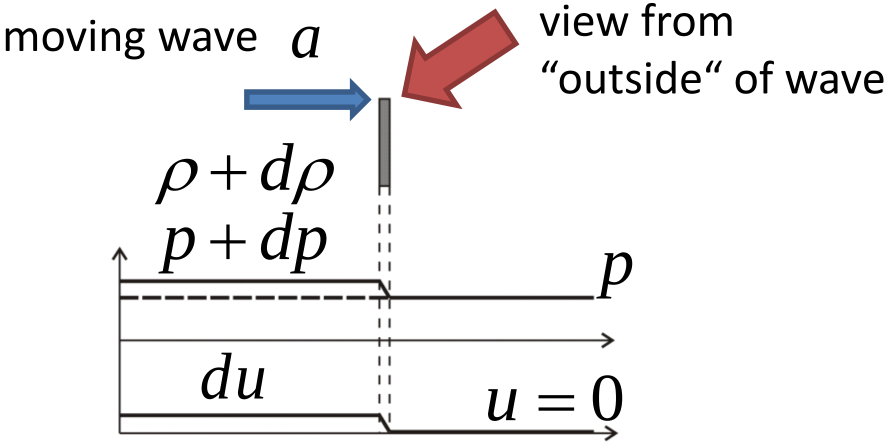
$p + dp, \rho + d\rho,$ $T + dT, du$

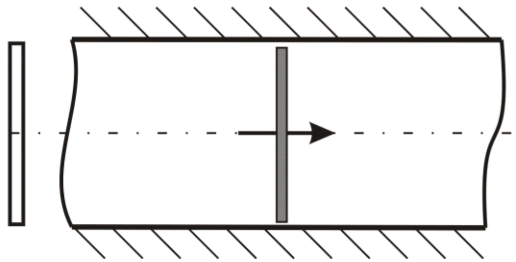
p, ρ, T $u = 0$

1. Fluid Flow and Thermodynamics

Speed of sound

- parameters of fluid





1. Fluid Flow and Thermodynamics

Speed of sound

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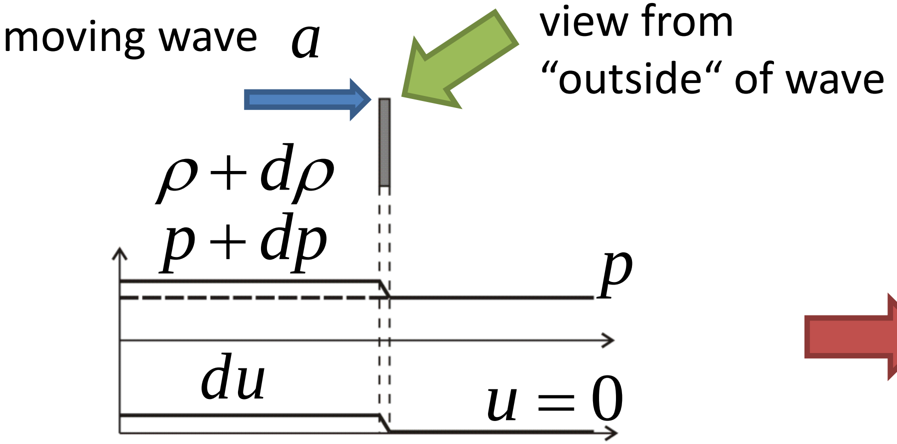
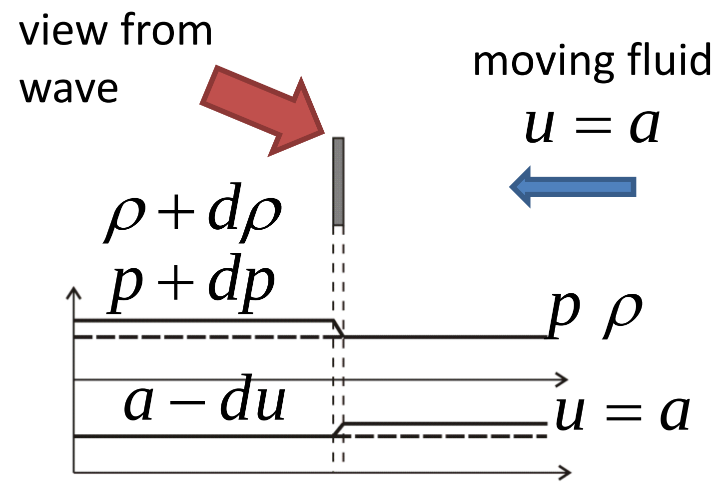
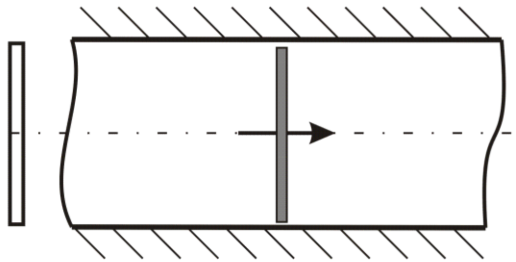


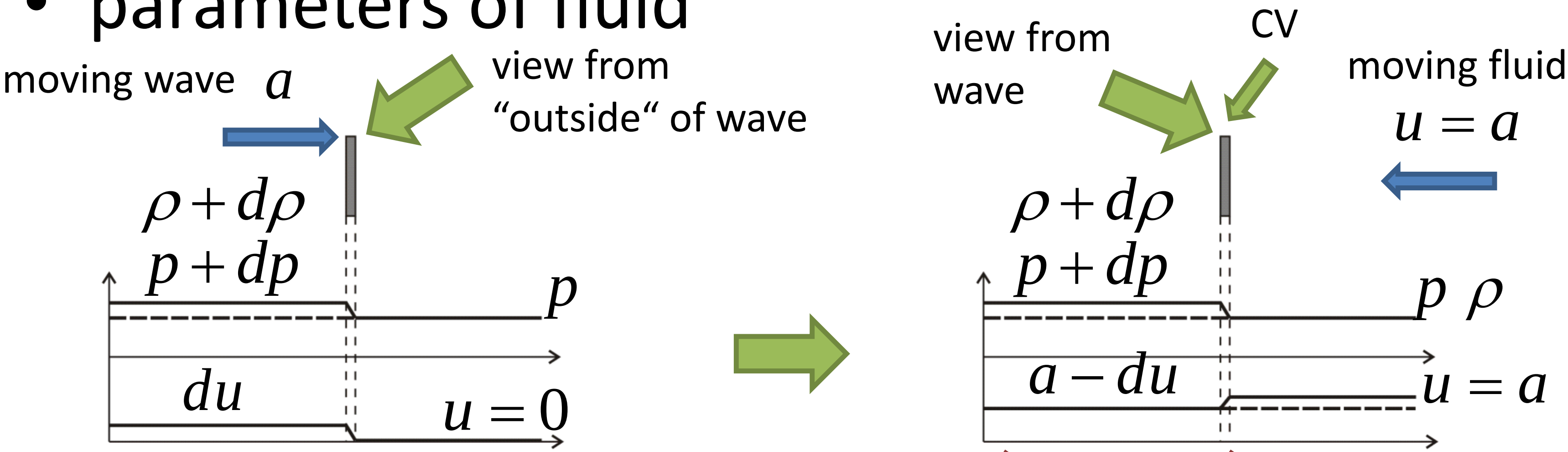




1. Fluid Flow and Thermodynamics

Speed of sound

- parameters of fluid



momentum equation

$$(\rho + d\rho)(a - du)\ A - \rho a^2 A =$$
$$= A\left(p - (p + dp)\right)$$

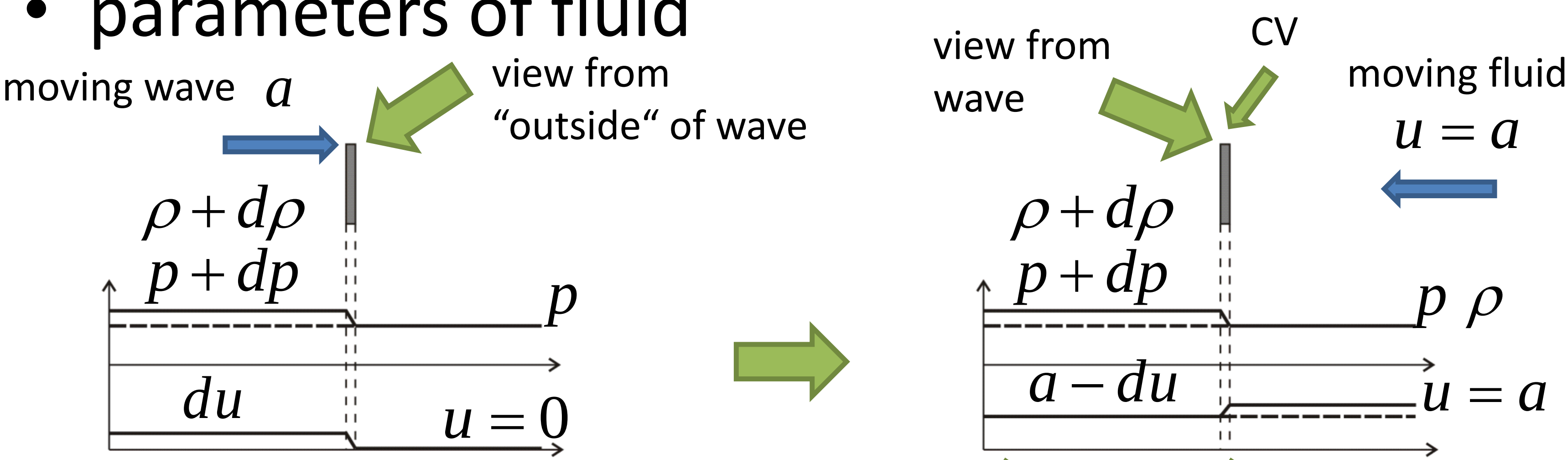
continuity equation

$$(\rho + d\rho)(a - du)A - \rho a A = 0$$

1. Fluid Flow and Thermodynamics

Speed of sound

- parameters of fluid



momentum equation

$$(\rho + d\rho)(a - du)\; A - \rho a^2 A =$$
$$= A(p - (p + dp))$$

$$a = \frac{dp}{\rho\, du}$$

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$$\rho\, du = a d\rho$$

1. Fluid Flow and Thermodynamics

Speed of sound

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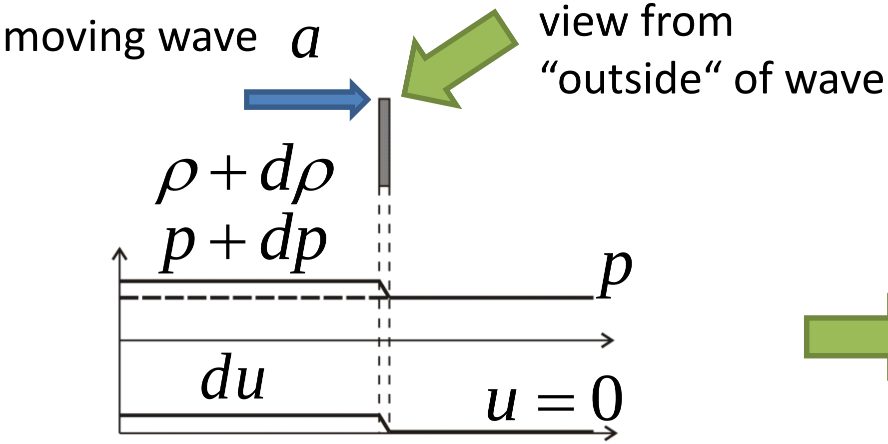

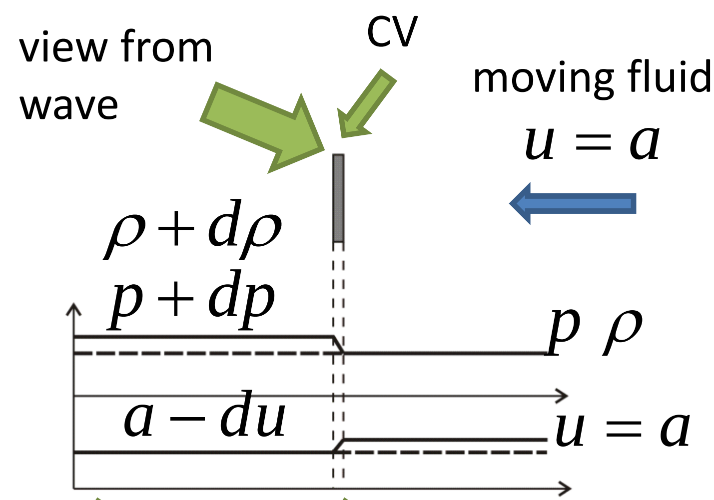

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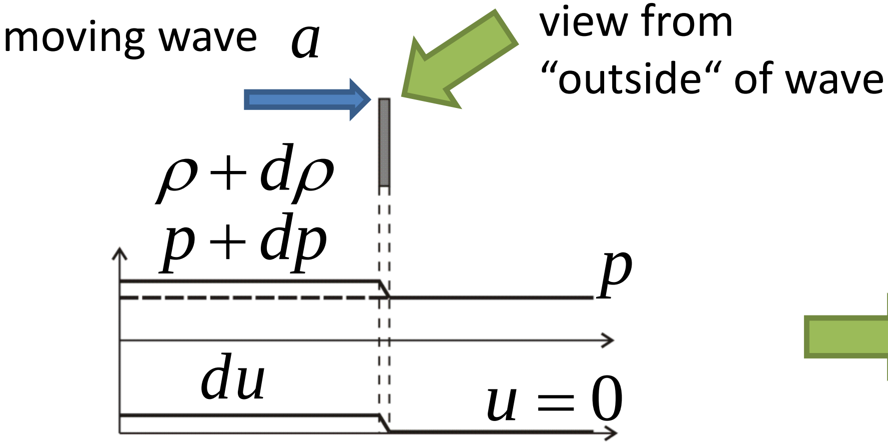
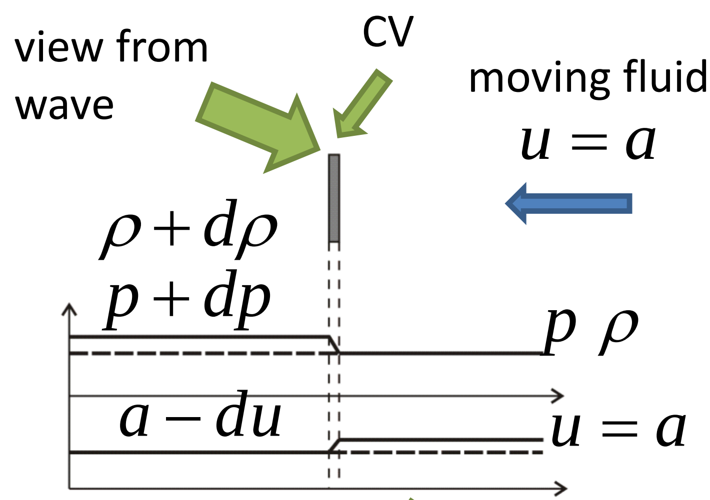
1. Fluid Flow and Thermodynamics

Speed of sound

- parameters of fluid





$$\frac{p}{p_1} = \left(\frac{\rho}{\rho_1}\right)^{\gamma}$$

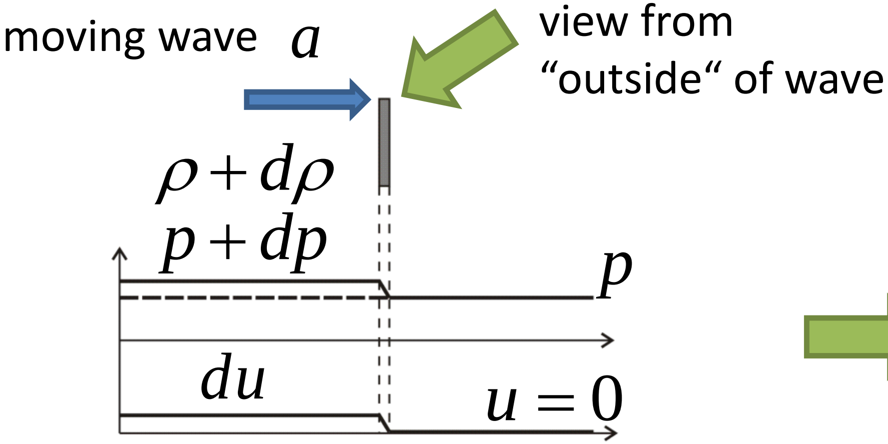
$$p = \frac{p_1}{\rho_1^{\gamma}} \rho^{\gamma}$$

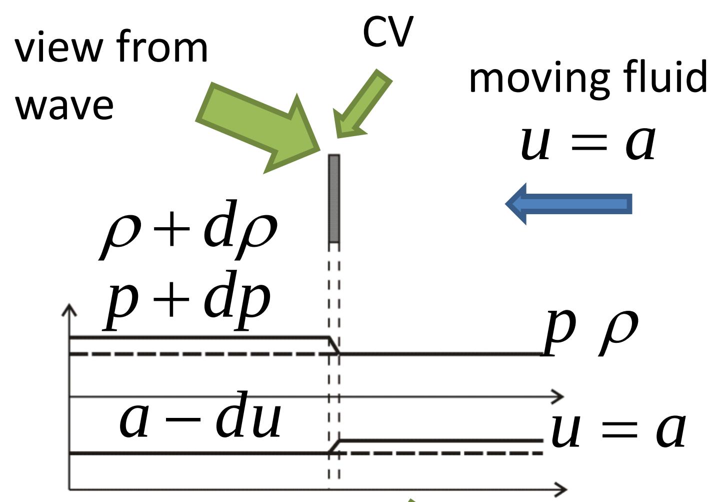
$$a^2 = \frac{dp}{d\rho}$$

1. Fluid Flow and Thermodynamics

Speed of sound

- parameters of fluid





$$\frac{p}{p_1} = \left(\frac{\rho}{\rho_1} \right)^{\gamma}$$

$$p = \frac{p_1}{\rho_1^{\gamma}} \rho^{\gamma}$$

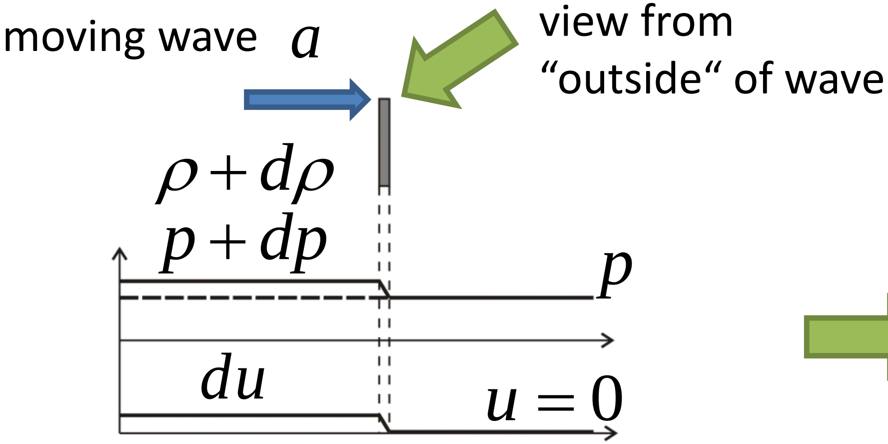
$$\frac{dp}{d\rho} = \frac{p_1}{\rho_1^{\gamma}} \gamma \rho^{\gamma - 1} = \frac{p}{\rho} \gamma = \gamma RT$$

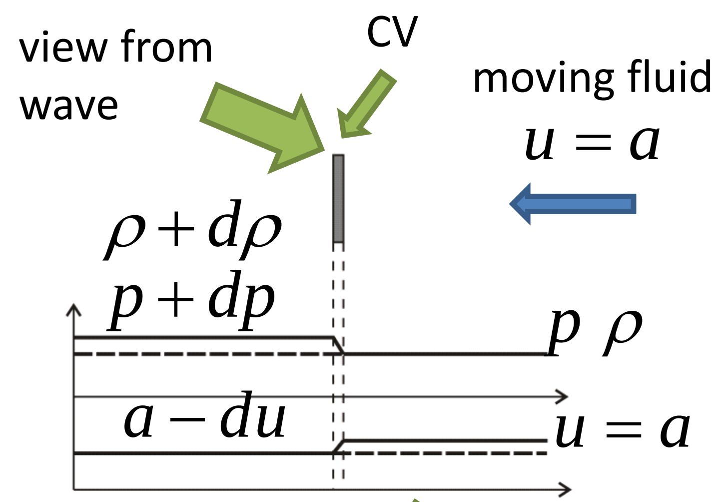
$$a^2 = \frac{dp}{d\rho}$$

1. Fluid Flow and Thermodynamics

Speed of sound

- parameters of fluid





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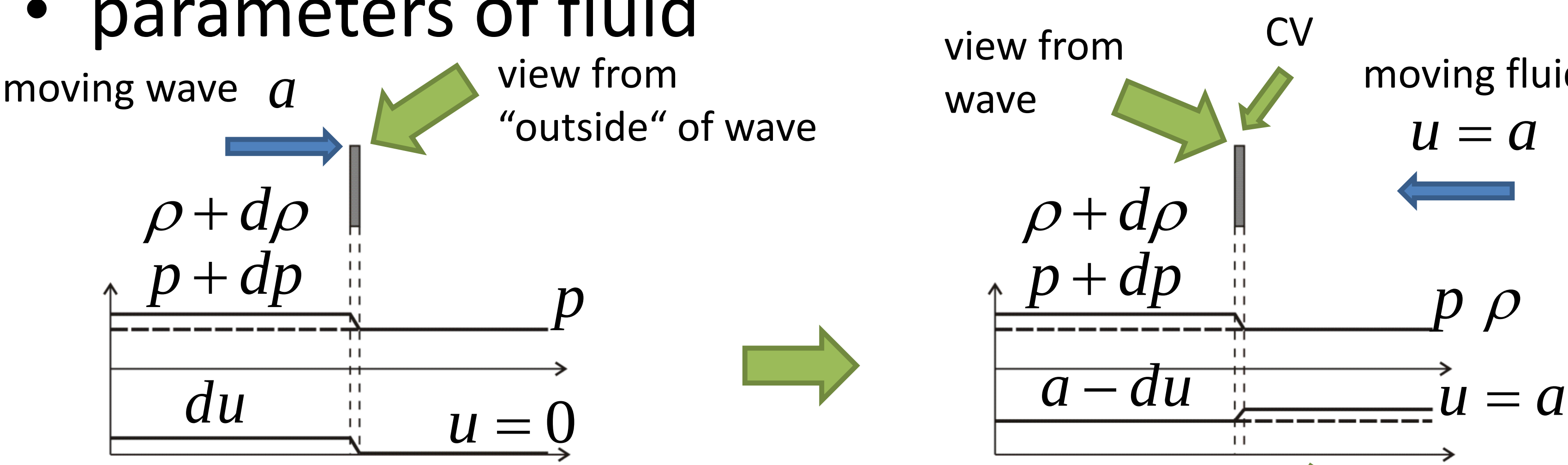
$$a^2 = \frac{dp}{d\rho}$$

$$a = \sqrt{\gamma RT}$$

1. Fluid Flow and Thermodynamics

Speed of sound

- parameters of fluid



$$\frac{p}{p_1} = \left(\frac{\rho}{\rho_1}\right)^{\gamma}$$

$$p = \frac{p_1}{\rho_1^{\gamma}} \rho^{\gamma}$$

$$\frac{dp}{d\rho} = \frac{p_1}{\rho_1^{\gamma}} \gamma \rho^{\gamma - 1} = \frac{p}{\rho} \gamma = \gamma RT$$

$$a^2 = \frac{dp}{d\rho}$$

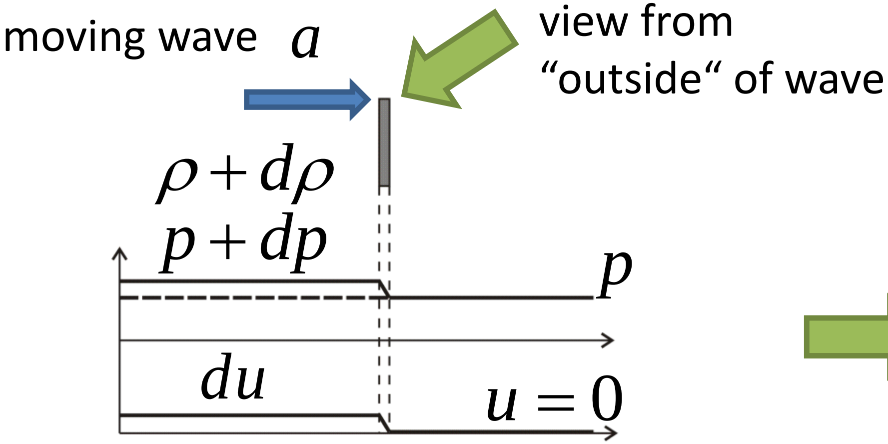
$$a = \sqrt{\gamma RT}$$

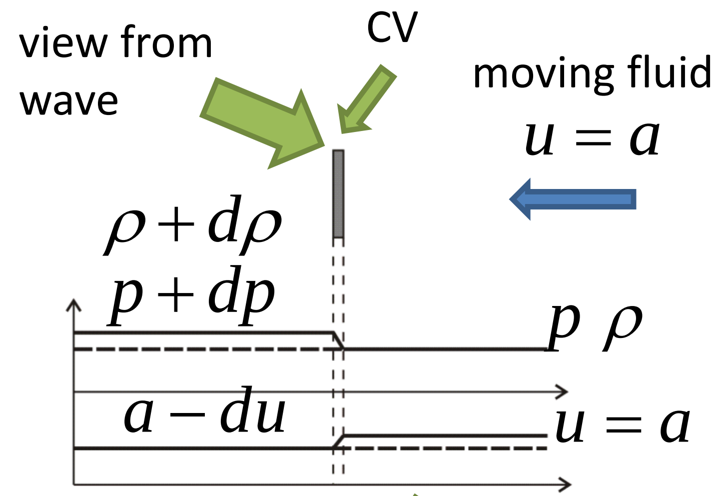
$$M = \frac{u}{a} = \frac{u}{\sqrt{\gamma RT}}$$

1. Fluid Flow and Thermodynamics

Speed of sound

- parameters of fluid





Gas	M_m [g/mol]	γ [–]	a at 0°C [m/s]
Air	28.96	1.404	331
Hydrogen	2.016	1.407	1270
Xenon	131.3	1.667	170

$$a^2 = \frac{dp}{d\rho}$$

$$a = \sqrt{\gamma \frac{R_M}{M_m} T}$$

$$a = \sqrt{\gamma RT}$$

$$M = \frac{u}{a} = \frac{u}{\sqrt{\gamma RT}}$$

1. Fluid Flow and Thermodynamics

Isentropic flow

- Importance of isentropic flow:
  - isentropic flow is adiabatic in which viscous losses are negligible
  - real flows are not isentropic

the effects of viscosity and heat transfer are restricted to thin layers near the walls

major part of the flow can be assumed to be isentropic

1. Fluid Flow and Thermodynamics

- Importance of isentropic flow:
  - isentropic flow is adiabatic in which viscous losses are negligible
  - real flows are not isentropic

the effects of viscosity and heat transfer are restricted to thin layers near the walls

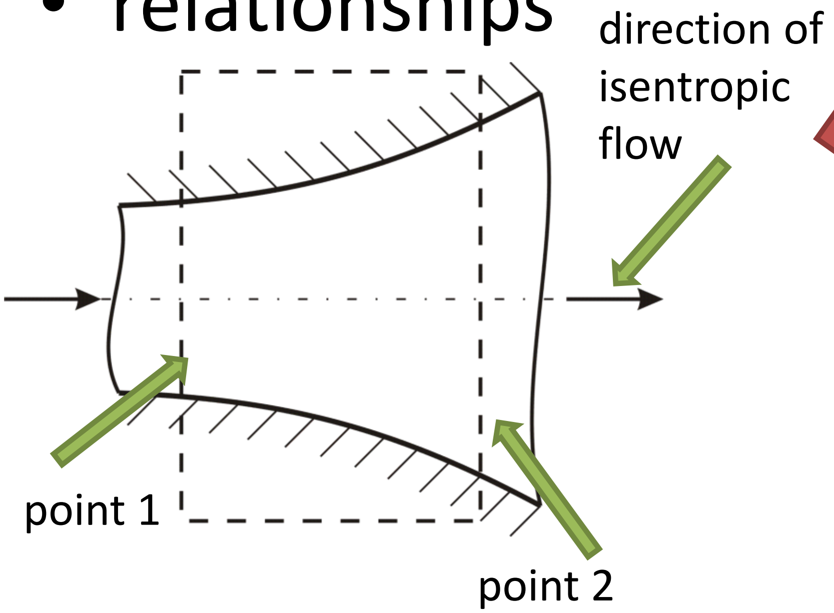
major part of the flow can be assumed to be isentropic

many flows in engineering practice can be adequately modeled by assuming them to be **isentropic** and also **steady-state** and **quasi-one dimensional flow**

1. Fluid Flow and Thermodynamics

Isentropic flow

- relationships



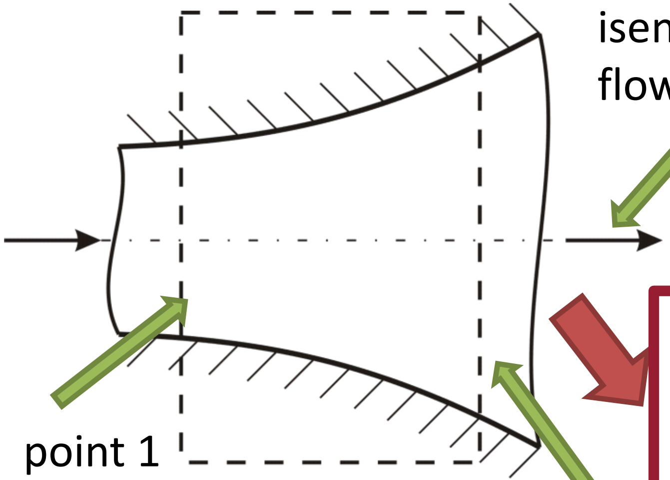
isentropic process 1-2

$$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma}$$

1. Fluid Flow and Thermodynamics

Isentropic flow

- relationships



isentropic process 1-2

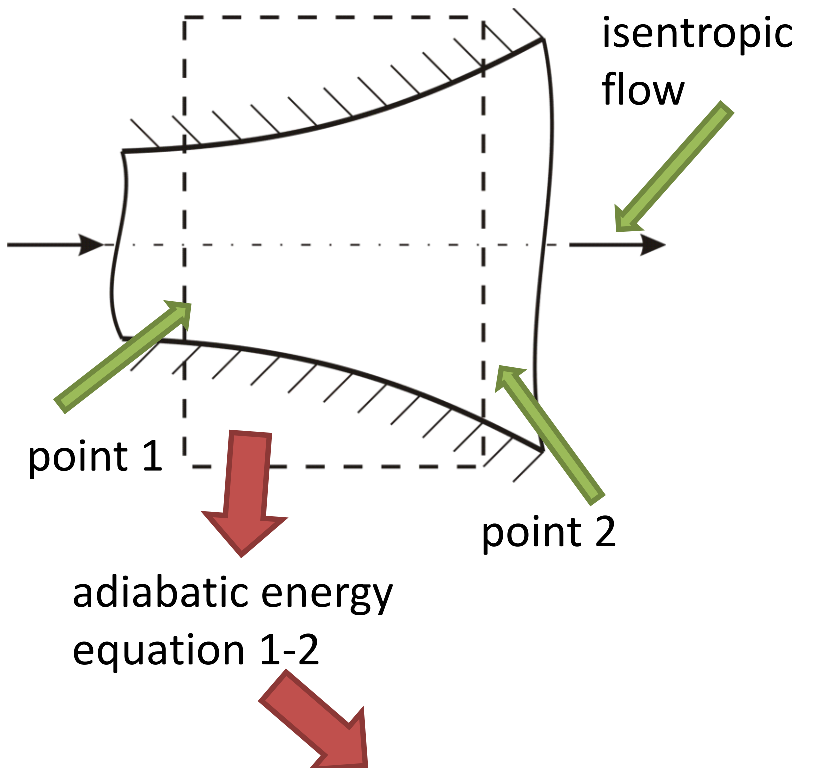
$$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma}$$

$$\frac{a_2}{a_1} = \frac{\sqrt{\gamma R T_2}}{\sqrt{\gamma R T_1}} = \left(\frac{T_2}{T_1}\right)^{\frac{1}{2}} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{2\gamma}} = \left(\frac{\rho_2}{\rho_1}\right)^{\frac{\gamma-1}{2}}$$

1. Fluid Flow and Thermodynamics

Isentropic flow

- relationships

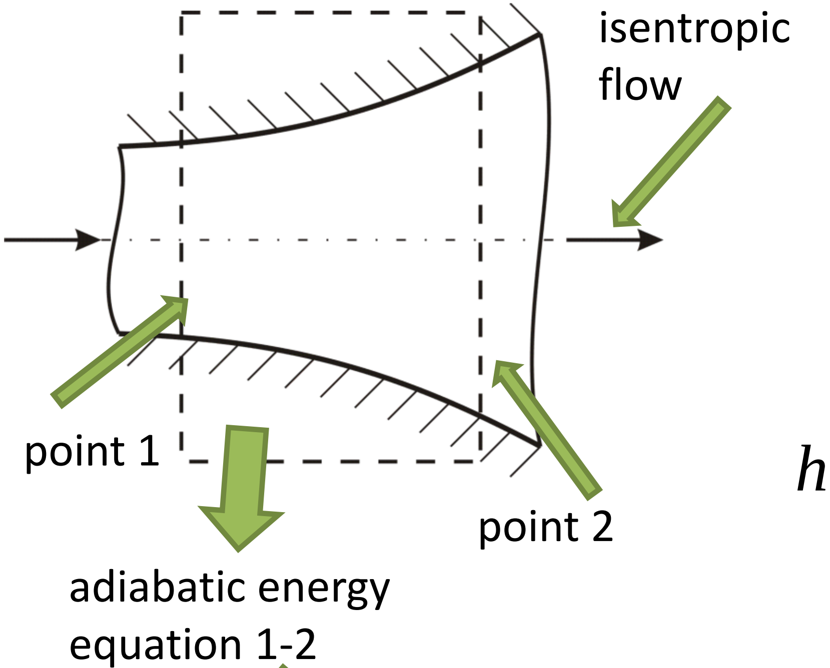


$$\left(h_1 + \frac{u_1^2}{2}\right) = \left(h_2 + \frac{u_2^2}{2}\right)$$

1. Fluid Flow and Thermodynamics

Isentropic flow

- relationships



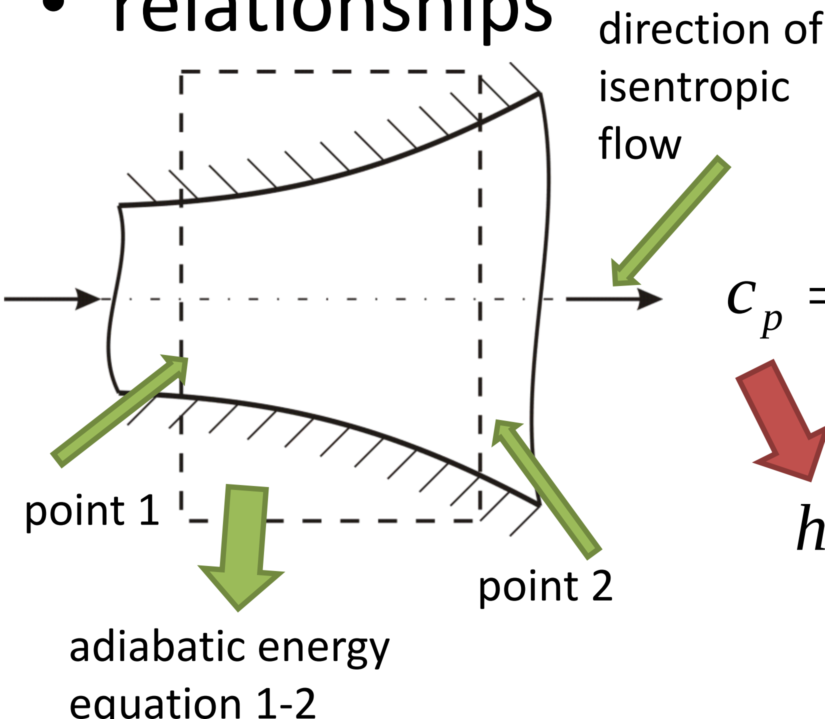
$$\left(h_1 + \frac{u_1^2}{2}\right) = \left(h_2 + \frac{u_2^2}{2}\right)$$

$$h = c_p T$$

1. Fluid Flow and Thermodynamics

Isentropic flow

- relationships



$$\left(h_1 + \frac{u_1^2}{2}\right) = \left(h_2 + \frac{u_2^2}{2}\right)$$

$$c_p = \frac{\gamma R}{\gamma - 1}$$

$$h = c_p T$$

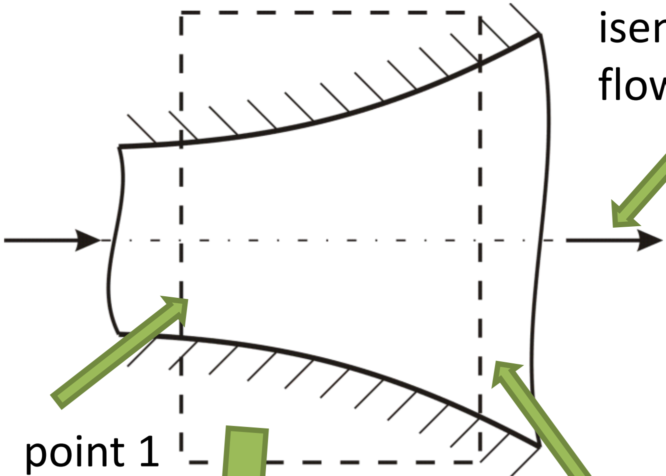
$$h = \frac{\gamma RT}{\gamma - 1}$$

1. Fluid Flow and Thermodynamics

Isentropic flow

- relationships

direction of isentropic flow



point 1

point 2

$$c_p = \frac{\gamma R}{\gamma - 1}$$

$$a = \sqrt{\gamma RT}$$

$$h = c_p T$$

$$h = \frac{\gamma RT}{\gamma - 1}$$

adiabatic energy equation 1-2

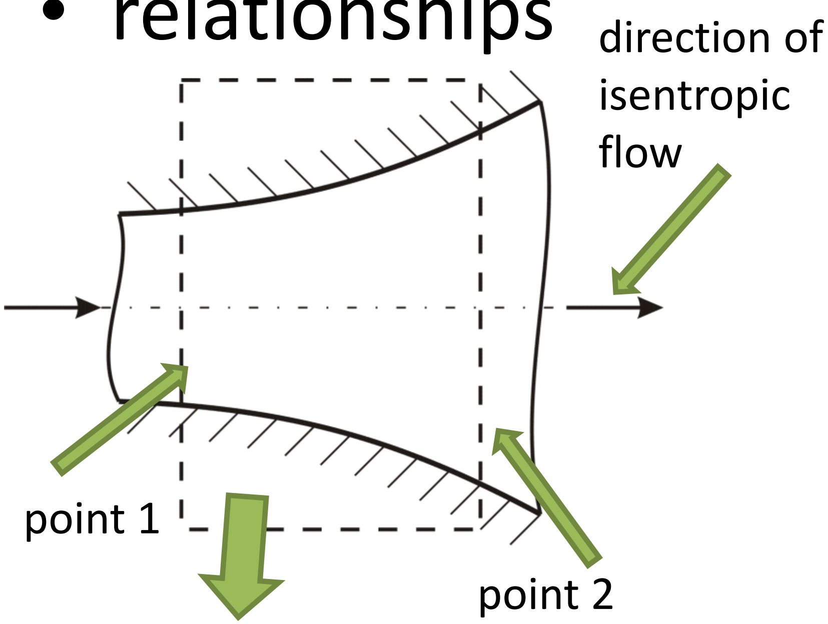
$$h = \frac{a^2}{\gamma - 1}$$

$$\left(h_1 + \frac{u_1^2}{2}\right) = \left(h_2 + \frac{u_2^2}{2}\right)$$

1. Fluid Flow and Thermodynamics

Isentropic flow

- relationships



adiabatic energy equation 1-2

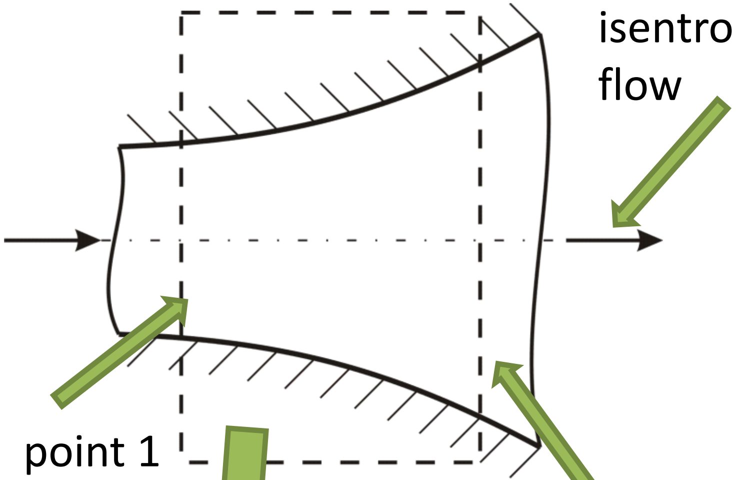
$$h = \frac{a^2}{\gamma - 1}$$

$$\left(h_1 + \frac{u_1^2}{2} \right) = \left(h_2 + \frac{u_2^2}{2} \right) \Rightarrow \left(\frac{a_1^2}{\gamma - 1} + \frac{u_1^2}{2} \right) = \left(\frac{a_2^2}{\gamma - 1} + \frac{u_2^2}{2} \right)$$

1. Fluid Flow and Thermodynamics

Isentropic flow

- relationships



adiabatic energy equation 1-2

$$h = \frac{a^2}{\gamma - 1}$$

$$\left(h_1 + \frac{u_1^2}{2}\right) = \left(h_2 + \frac{u_2^2}{2}\right) \Rightarrow \left(\frac{a_1^2}{\gamma - 1} + \frac{u_1^2}{2}\right) = \left(\frac{a_2^2}{\gamma - 1} + \frac{u_2^2}{2}\right)$$

critical point

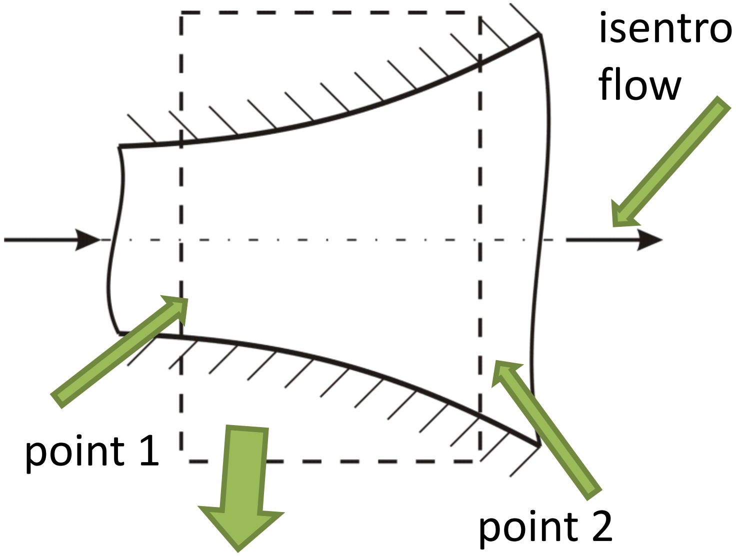
$$\left(\frac{a^2}{\gamma - 1} + \frac{u^2}{2}\right) = \frac{\gamma + 1}{2(\gamma - 1)} a^{*2}$$

$$\left(\frac{a^2}{u^2(\gamma - 1)} + \frac{1}{2}\right) = \frac{\gamma + 1}{2(\gamma - 1)} \frac{a^{*2}}{u^2}$$

1. Fluid Flow and Thermodynamics

Isentropic flow

- relationships



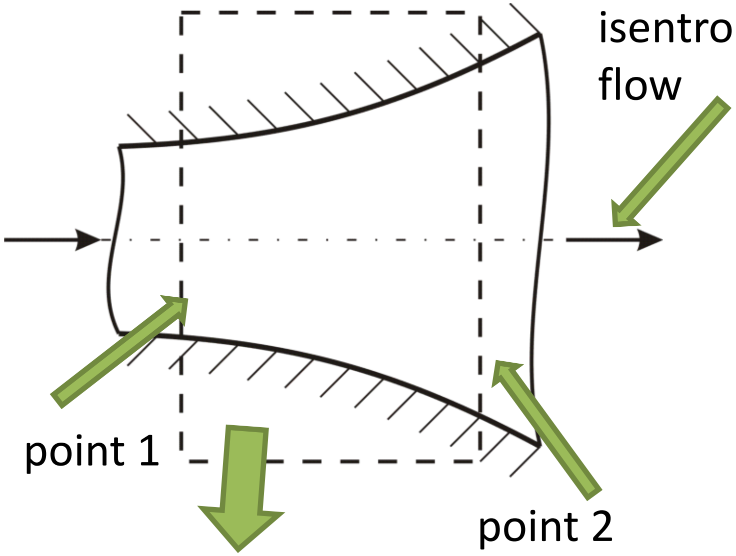
$$\left(\frac{a^2}{u^2(\gamma-1)}+\frac{1}{2}\right)=\frac{\gamma+1}{2(\gamma-1)}\frac{a^{*2}}{u^2}$$

$$\left(\frac{1}{M^2(\gamma-1)}+\frac{1}{2}\right)=\frac{\gamma+1}{2(\gamma-1)}\frac{1}{\left(M^*\right)^2}$$

1. Fluid Flow and Thermodynamics

Isentropic flow

- relationships



$$\left(\frac{a^2}{u^2(\gamma-1)}+\frac{1}{2}\right)=\frac{\gamma+1}{2(\gamma-1)}\frac{a^{*2}}{u^2}$$

$$\left(\frac{1}{M^2(\gamma-1)}+\frac{1}{2}\right)=\frac{\gamma+1}{2(\gamma-1)}\frac{1}{(M^*)^2}$$

$$(M^*)^2=\frac{(\gamma+1)M^2}{2+(\gamma-1)M^2}$$

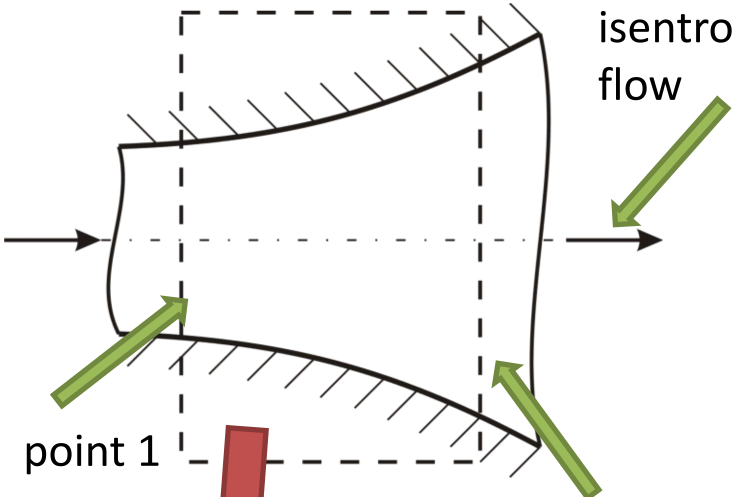
$$M^2=\frac{2}{(\gamma+1)/(M^*)^2-(\gamma-1)}$$

1. Fluid Flow and Thermodynamics

Isentropic flow

- relationships

direction of isentropic flow



point 1

point 2

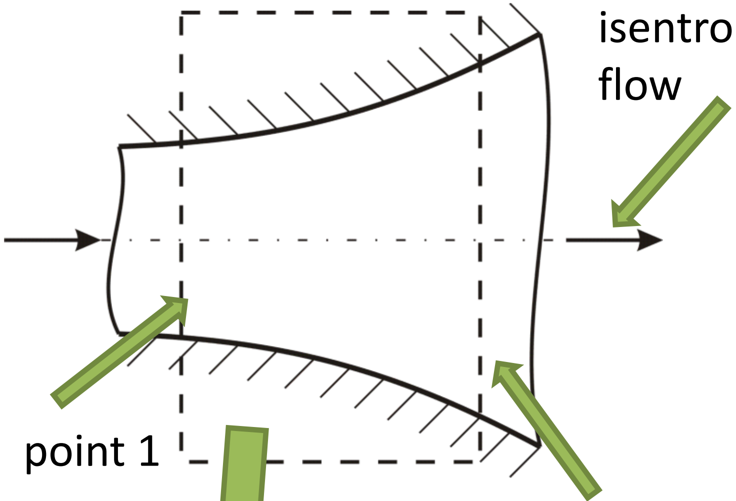
adiabatic energy equation

$$\left(h+\frac{u^2}{2}\right)=h_0 \Rightarrow \left(c_pT+\frac{u^2}{2}\right)=c_pT_0$$

1. Fluid Flow and Thermodynamics

Isentropic flow

- relationships



adiabatic energy equation

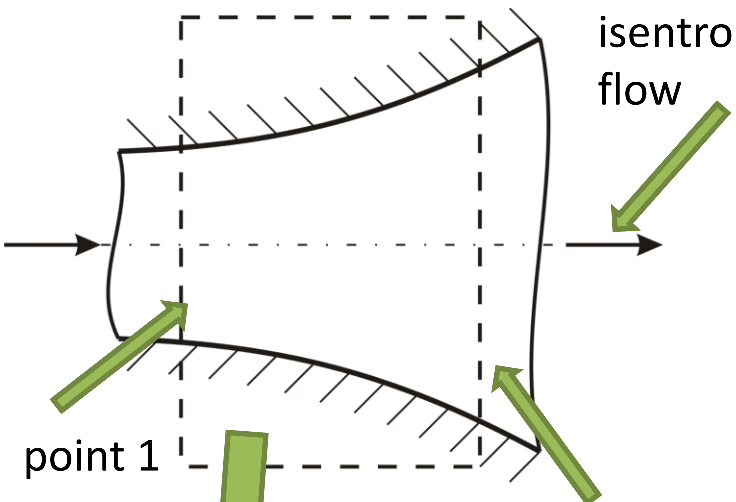
$$\left(h + \frac{u^2}{2}\right) = h_0 \Rightarrow \left(c_p T + \frac{u^2}{2}\right) = c_p T_0$$

$$1 + \frac{u^2}{2 c_p T} = \frac{T_0}{T}$$

1. Fluid Flow and Thermodynamics

Isentropic flow

- relationships



adiabatic energy equation

$$\left(h+\frac{u^2}{2}\right)=h_0$$

$$\left(c_pT+\frac{u^2}{2}\right)=c_pT_0$$

$$1+\frac{u^2}{2c_pT}=\frac{T_0}{T}$$

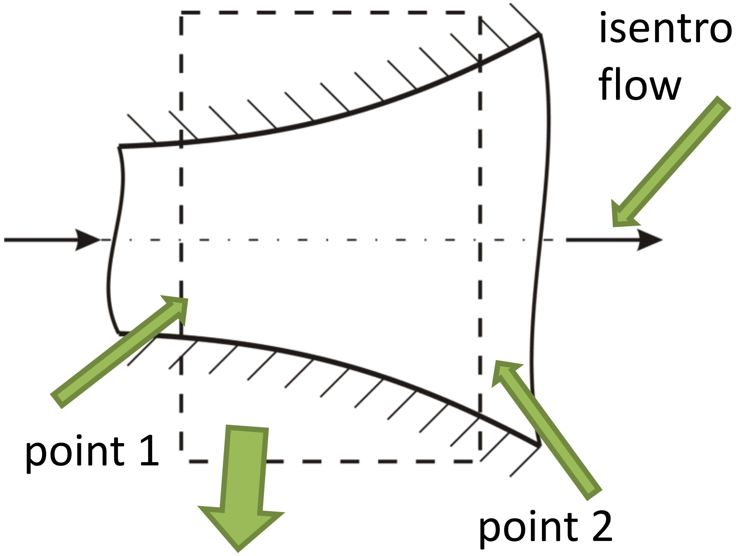
$$c_p=\frac{\gamma R}{\gamma-1}$$

$$1+\frac{(\gamma-1)u^2}{2\gamma RT}=\frac{T_0}{T}$$

1. Fluid Flow and Thermodynamics

Isentropic flow

- relationships



adiabatic energy equation

$$\left(h+\frac{u^2}{2}\right)=h_0 \quad\Rightarrow\quad \left(c_pT+\frac{u^2}{2}\right)=c_pT_0$$

$$1+\frac{u^2}{2c_pT}=\frac{T_0}{T} \qquad c_p=\frac{\gamma R}{\gamma-1}$$

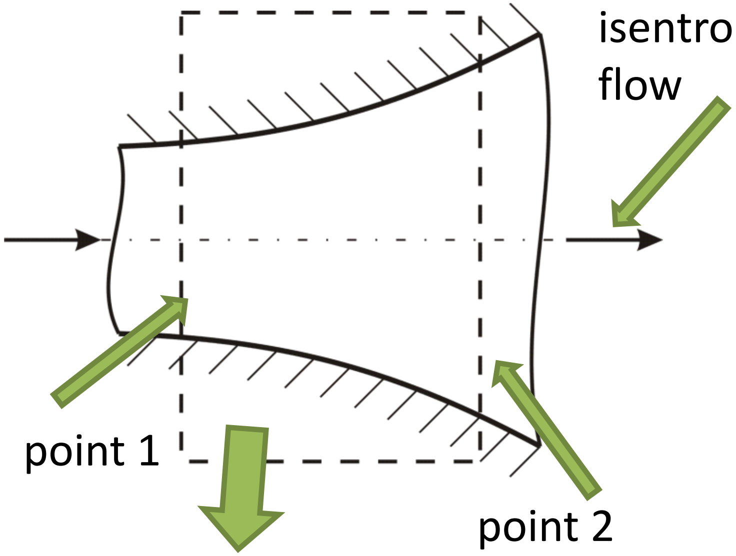
$$1+\frac{(\gamma-1)u^2}{2\gamma RT}=\frac{T_0}{T}$$

$$1+\frac{(\gamma-1)}{2}M^2=\frac{T_0}{T}$$

1. Fluid Flow and Thermodynamics

Isentropic flow

- relationships



$$\left(1+\frac{(\gamma-1)}{2}M^2\right)^{\frac{1}{\gamma-1}}=\frac{\rho_0}{\rho}$$

$$\left(1+\frac{(\gamma-1)}{2}M^2\right)^{\frac{\gamma}{\gamma-1}}=\frac{p_0}{p}$$

$$1+\frac{(\gamma-1)}{2}M^2=\frac{T_0}{T}$$

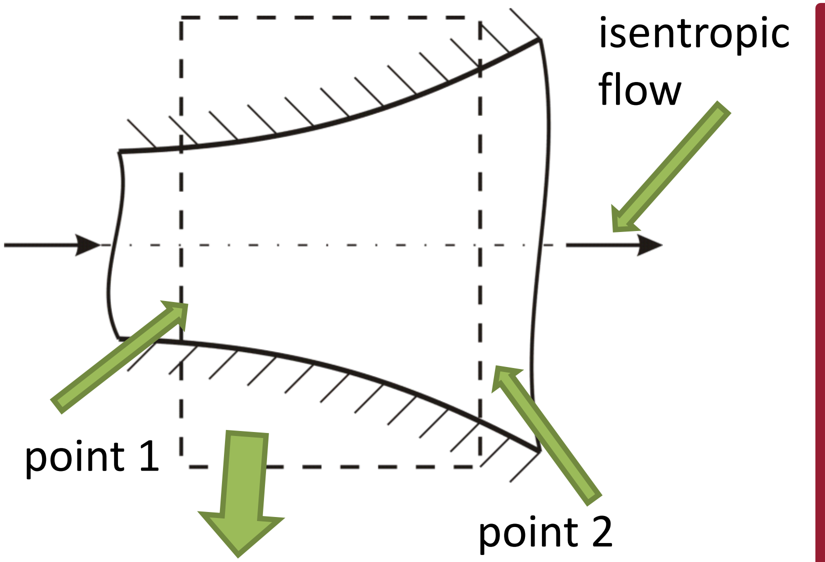
$$\left(h+\frac{u^2}{2}\right)=h_0 \Rightarrow \left(c_pT+\frac{u^2}{2}\right)=c_pT_0$$

$$\frac{p_0}{p}=\left(\frac{T_0}{T}\right)^{\frac{\gamma}{\gamma-1}}=\left(\frac{\rho_0}{\rho}\right)^{\gamma}$$

1. Fluid Flow and Thermodynamics

Isentropic flow

- relationships



$$\left(1+\frac{(\gamma-1)}{2}M^2\right)^{\frac{1}{\gamma-1}}=\frac{\rho_0}{\rho}$$

$$\left(1+\frac{(\gamma-1)}{2}M^2\right)^{\frac{\gamma}{\gamma-1}}=\frac{p_0}{p}$$

$$1+\frac{(\gamma-1)}{2}M^2=\frac{T_0}{T}$$

if $M=1$

$$\rho=\rho^*$$

$$p=p^*$$

$$T=T^*$$

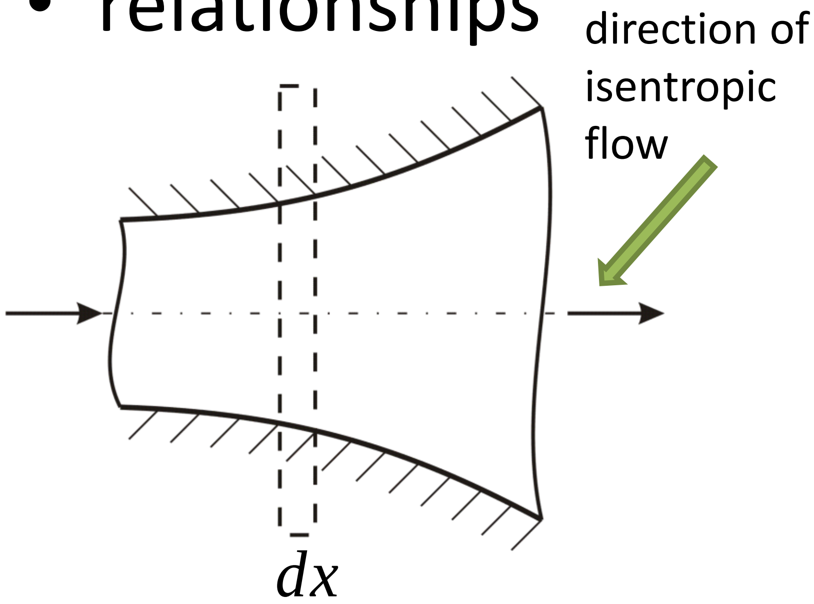
$$\left(h+\frac{u^2}{2}\right)=h_0 \;\Rightarrow\; \left(c_pT+\frac{u^2}{2}\right)=c_pT_0$$

$$\frac{p_0}{p}=\left(\frac{T_0}{T}\right)^{\frac{\gamma}{\gamma-1}}=\left(\frac{\rho_0}{\rho}\right)^{\gamma}$$

1. Fluid Flow and Thermodynamics

Isentropic flow

- relationships



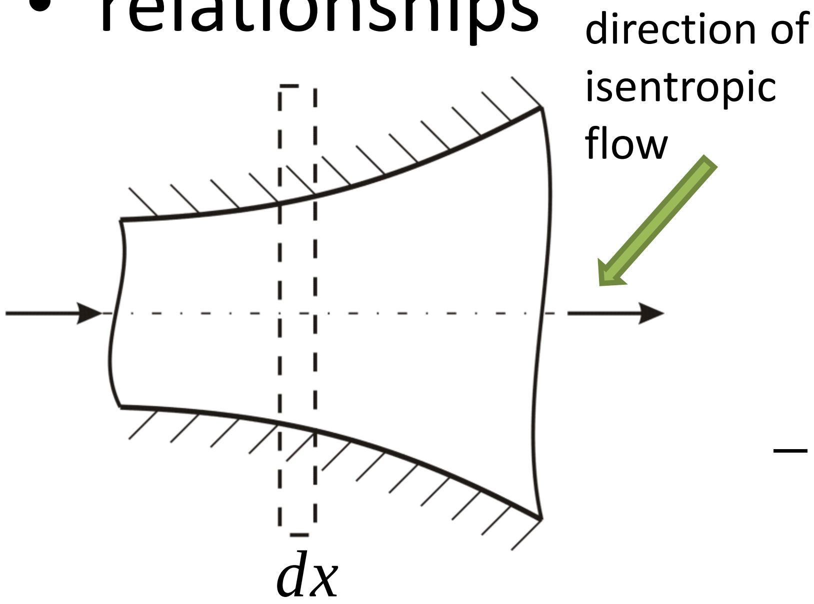
momentum equation

$$-\frac{dp}{\rho} = udu$$

1. Fluid Flow and Thermodynamics

Isentropic flow

- relationships



momentum equation

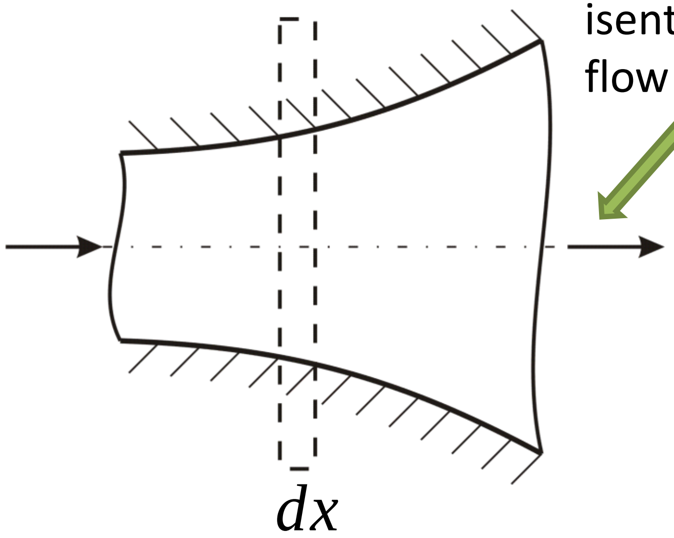
$$-\frac{dp}{\rho}=udu$$

$$-\frac{dp}{p}=\frac{\rho u}{pu}udu$$

1. Fluid Flow and Thermodynamics

Isentropic flow

- relationships



direction of isentropic flow

momentum equation

$$-\frac{dp}{\rho} = udu$$

speed of sound

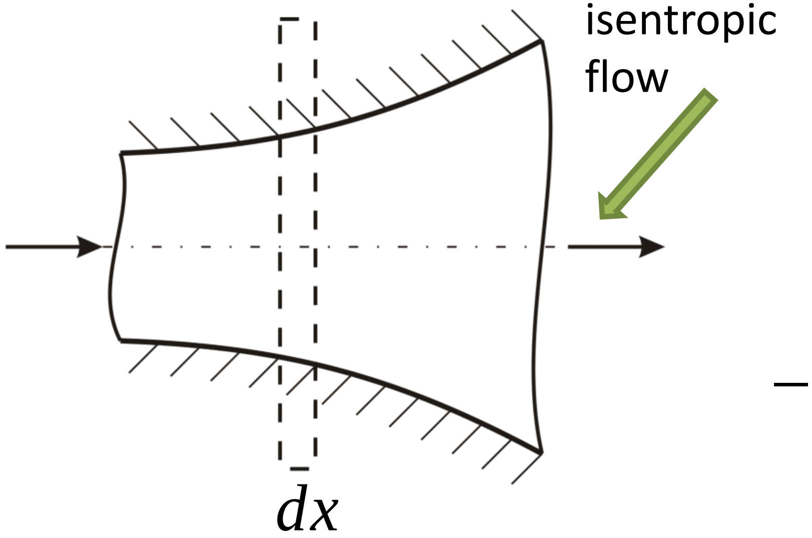
$$a^2 = \gamma RT = \gamma \frac{p}{\rho}$$

$$-\frac{dp}{p} = \frac{\rho u}{pu} udu$$

1. Fluid Flow and Thermodynamics

Isentropic flow

- relationships



momentum equation

$$-\frac{dp}{\rho} = udu$$

speed of sound

$$a^2 = \gamma RT = \gamma \frac{p}{\rho}$$

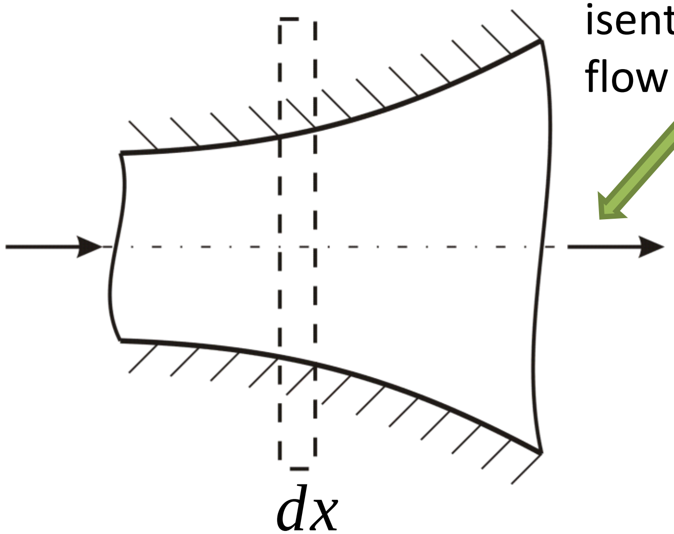
$$-\frac{dp}{p} = \frac{\rho u}{pu} udu$$

$$\frac{dp}{p} = -\gamma \frac{u^2}{a^2} \frac{du}{u}$$

1. Fluid Flow and Thermodynamics

Isentropic flow

- relationships



momentum equation

$$-\frac{dp}{\rho} = udu$$

speed of sound

$$a^2 = \gamma RT = \gamma \frac{p}{\rho}$$

$$-\frac{dp}{p} = \frac{\rho u}{pu} udu$$

$$\frac{dp}{p} = -\gamma \frac{u^2}{a^2} \frac{du}{u}$$

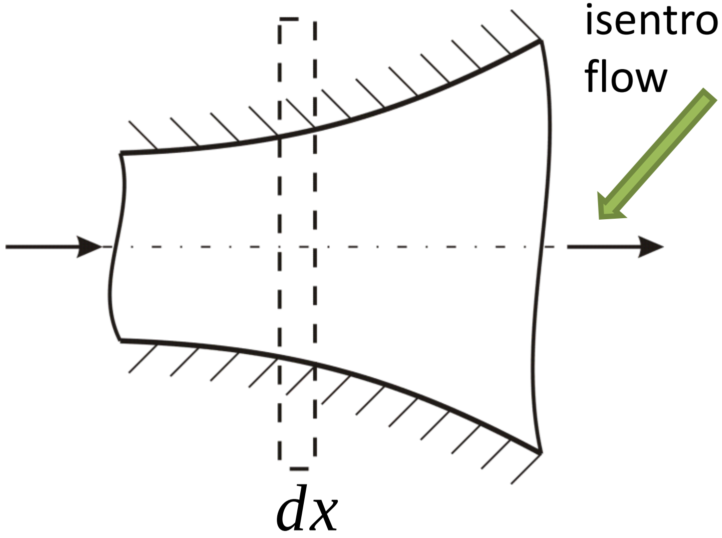
$$\frac{dp}{p} = -\gamma M^2 \frac{du}{u}$$

- magnitude of fractional pressure change induced by a given fractional velocity change depends on square of Mach number

1. Fluid Flow and Thermodynamics

Isentropic flow

- relationships



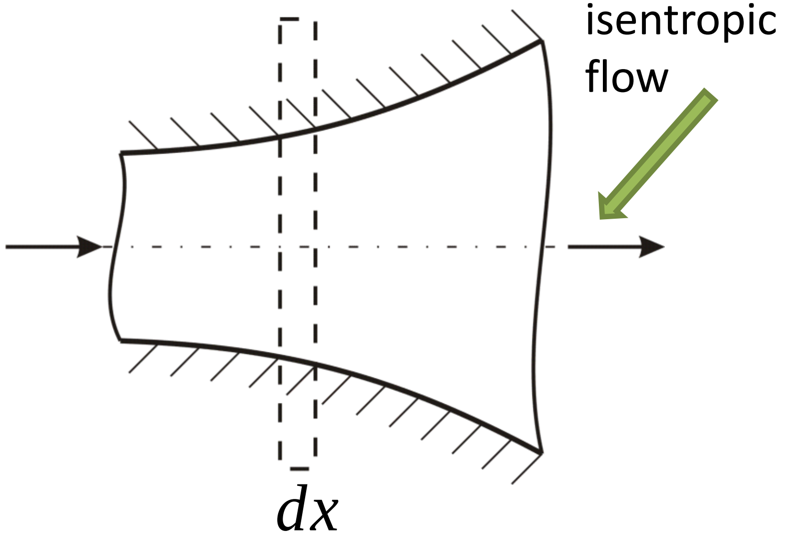
energy equation

$$dh + udu = 0$$

1. Fluid Flow and Thermodynamics

Isentropic flow

- relationships



energy equation

$$dh + udu = 0$$

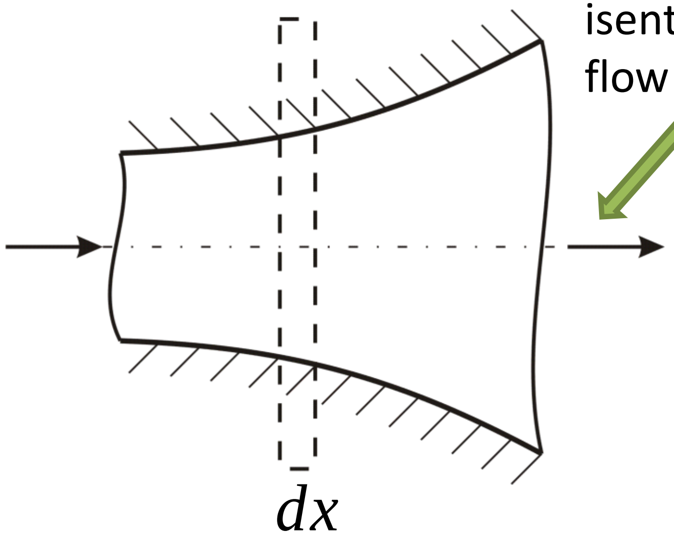
calorically perfect gas

$$dh = c_p dT$$

1. Fluid Flow and Thermodynamics

Isentropic flow

- relationships



direction of isentropic flow

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1. Fluid Flow and Thermodynamics

Isentropic flow

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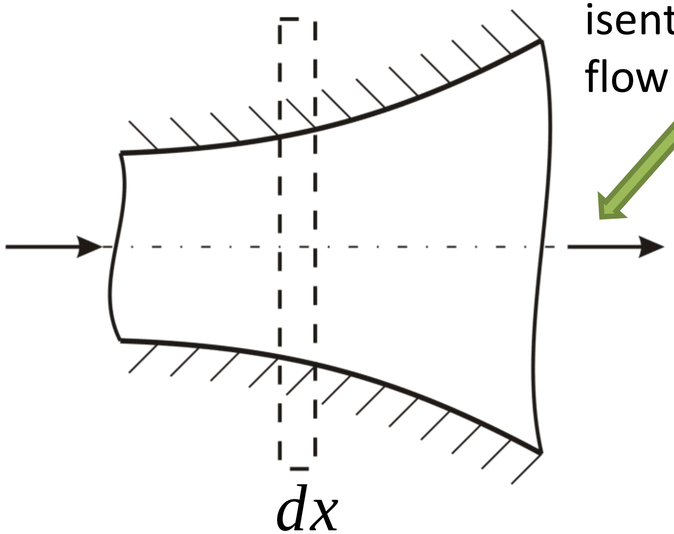

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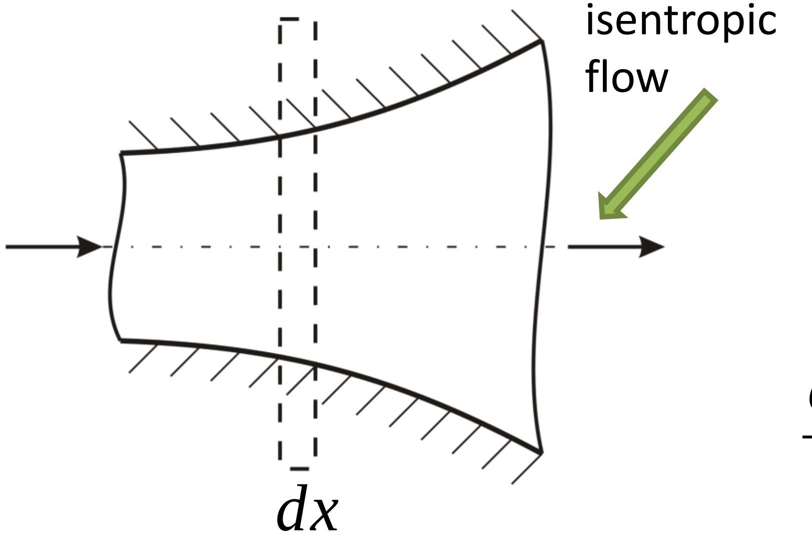
1. Fluid Flow and Thermodynamics

Isentropic flow

- relationships



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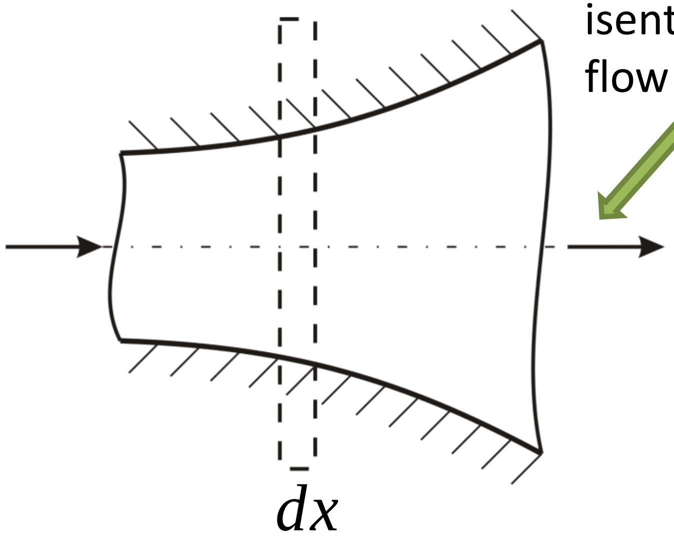
speed of sound

$$a^2 = \gamma RT = \gamma \frac{p}{\rho}$$

1. Fluid Flow and Thermodynamics

Isentropic flow

- relationships



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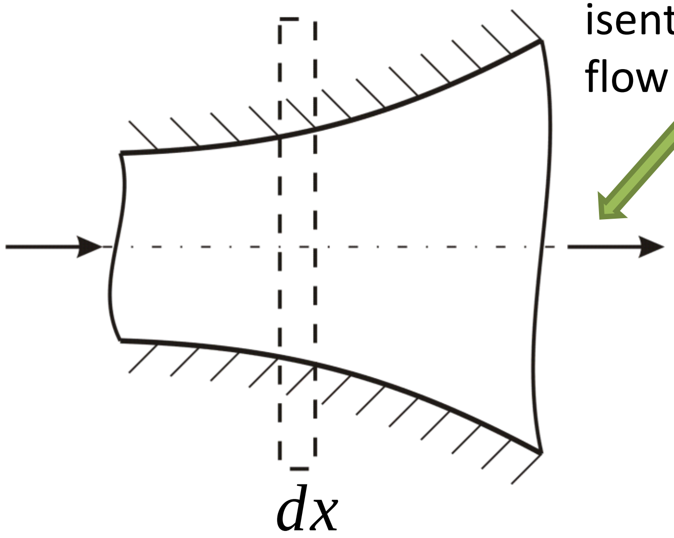
1. Fluid Flow and Thermodynamics

Isentropic flow

- relationships



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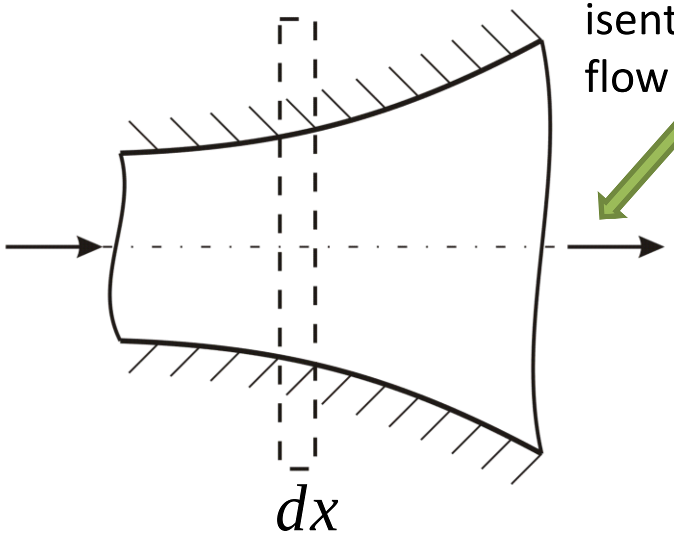
1. Fluid Flow and Thermodynamics

Isentropic flow

- relationships



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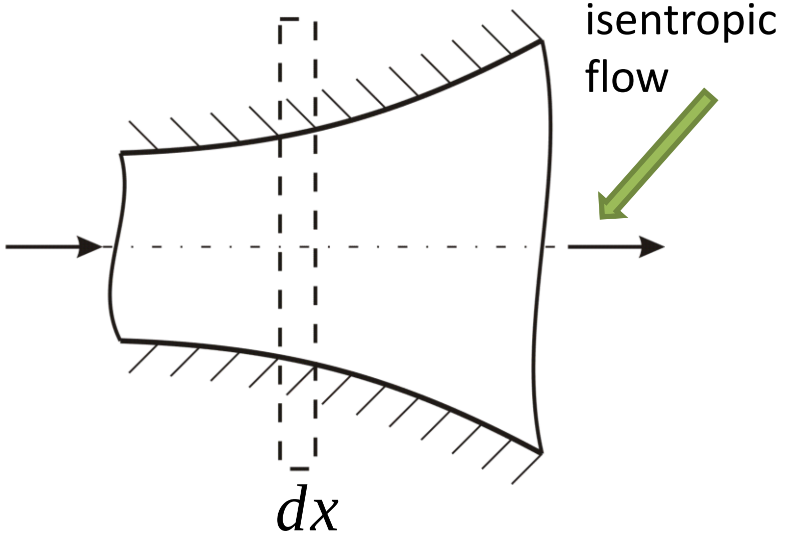
$$\frac{dT}{T} = -(\gamma - 1) M^2 \frac{du}{u}$$

- magnitude of fractional temperature change induced by a given fractional velocity change depends on square of Mach number

1. Fluid Flow and Thermodynamics

Isentropic flow

- relationships



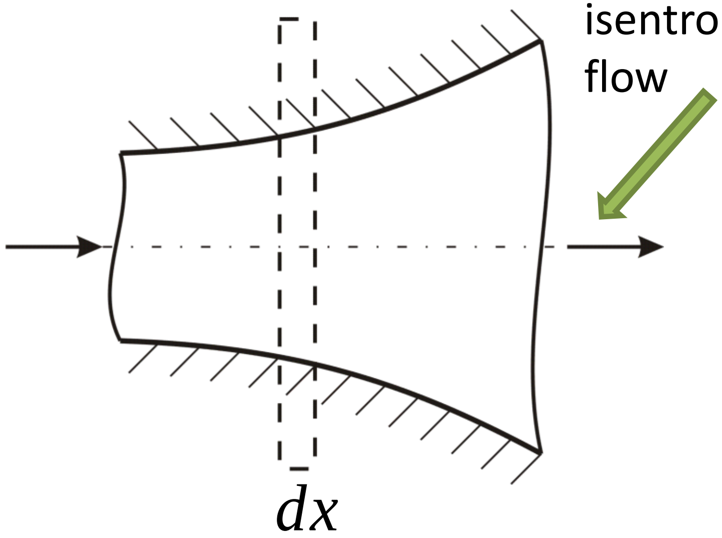
equation of state

$$\frac{p}{\rho T} = R$$

1. Fluid Flow and Thermodynamics

Isentropic flow

- relationships



equation of state

$$\frac{p}{\rho T} = R$$

$$\frac{dp}{p} - \frac{d\rho}{\rho} - \frac{dT}{T} = 0$$

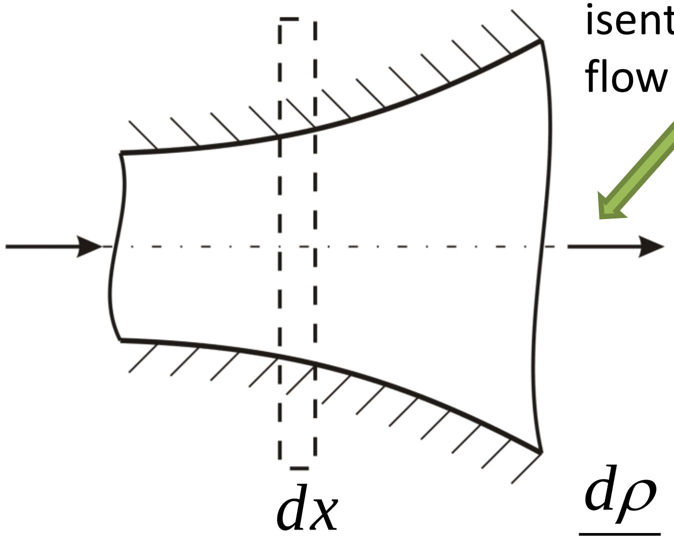
$$\frac{dT}{T} = -(\gamma - 1) M^2 \frac{du}{u}$$

$$\frac{dp}{p} = -\gamma M^2 \frac{du}{u}$$

1. Fluid Flow and Thermodynamics

Isentropic flow

- relationships



direction of isentropic flow

equation of state

$$\frac{p}{\rho T} = R$$

$$\frac{dT}{T} = -(\gamma - 1) M^2 \frac{du}{u}$$

$$\frac{dp}{p} - \frac{d\rho}{\rho} - \frac{dT}{T} = 0$$

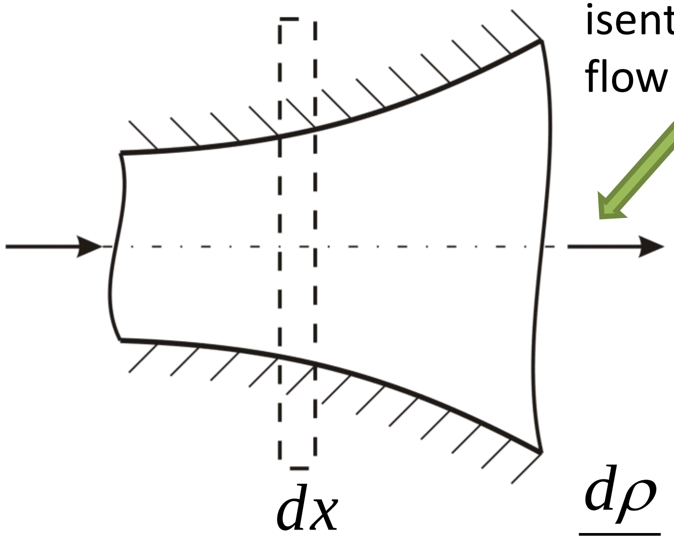
$$\frac{dp}{p} = -\gamma M^2 \frac{du}{u}$$

$$\frac{d\rho}{\rho} = -\gamma M^2 \frac{du}{u} + (\gamma - 1) M^2 \frac{du}{u}$$

1. Fluid Flow and Thermodynamics

Isentropic flow

- relationships



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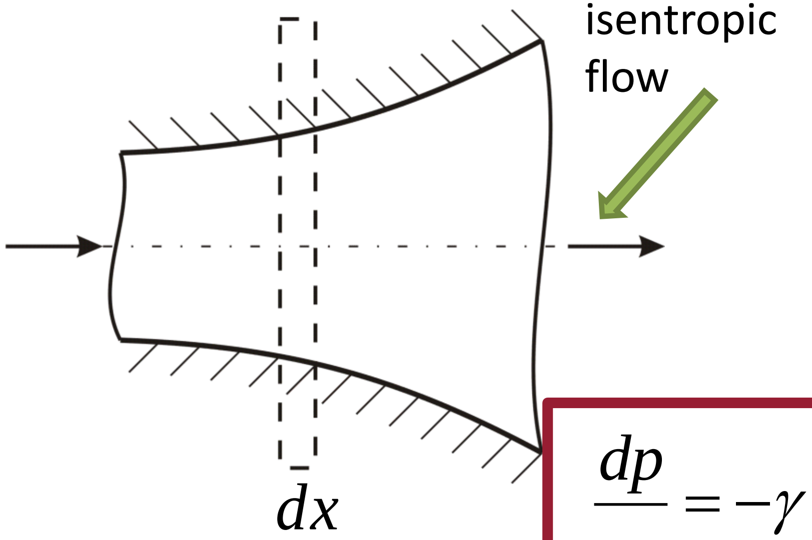
1. Fluid Flow and Thermodynamics

Isentropic flow

- relationships



- magnitude of fractional properties change induced by a given fractional velocity change depends on square of Mach number

$$\frac{dp}{p} = -\gamma M^2 \frac{du}{u}$$

$$\frac{dT}{T} = -(\gamma - 1) M^2 \frac{du}{u}$$

$$\frac{d\rho}{\rho} = -M^2 \frac{du}{u}$$

$$\frac{dp/p}{du/u} = -\gamma M^2$$

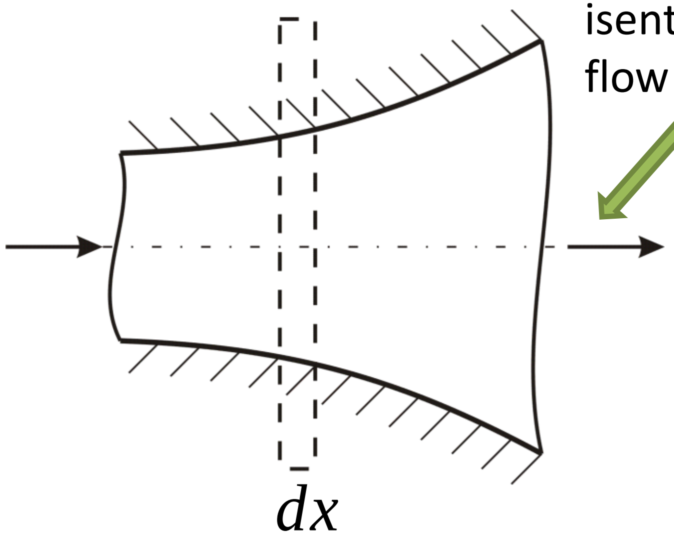
$$\frac{dT/T}{du/u} = -(\gamma - 1) M^2$$

$$\frac{d\rho/\rho}{du/u} = -M^2$$

1. Fluid Flow and Thermodynamics

Isentropic flow

- relationships



direction of isentropic flow

- magnitude of fractional properties change induced by a given fractional velocity change depends on square of Mach number

$$\frac{dp/p}{du/u} = -\gamma M^2$$

$$\frac{dT/T}{du/u} = -(\gamma - 1)M^2$$

$$\frac{d\rho/\rho}{du/u} = -M^2$$

	fractional propert. change induced by fractional velocity change of air [%]		
Mach num.	density	temp.	pressure
0.1	1	1.4	0.4
0.33	10.9	15.2	4.4
0.4	16	22.4	6.4

1. Fluid Flow and Thermodynamics

- governing equations for analysis of nozzle

continuity

$$\frac{d\rho}{\rho}+\frac{du}{u}+\frac{dA}{A}=0$$

momentum

$$-\frac{dp}{\rho}=udu$$

energy

$$dh+udu=0$$

state

$$\frac{dp}{p}-\frac{d\rho}{\rho}-\frac{dT}{T}=0$$

1. Fluid Flow and Thermodynamics

- governing equations for analysis of nozzle

$$\frac{d\rho}{\rho}+\frac{du}{u}+\frac{dA}{A}=0$$

$$\frac{d\rho}{\rho}=-M^2\frac{du}{u}$$

$$-\frac{dp}{\rho}=udu$$

$$dh+udu=0$$

$$\frac{dp}{p}-\frac{d\rho}{\rho}-\frac{dT}{T}=0$$

1. Fluid Flow and Thermodynamics

- governing equations for analysis of nozzle

$$\frac{d\rho}{\rho}+\frac{du}{u}+\frac{dA}{A}=0 \quad \Rightarrow \quad -M^2\frac{du}{u}+\frac{du}{u}+\frac{dA}{A}=0$$

$$\frac{d\rho}{\rho}=-M^2\frac{du}{u} \quad \Leftarrow \quad -\frac{dp}{\rho}=udu$$

$$dh+udu=0$$

$$\frac{dp}{p}-\frac{d\rho}{\rho}-\frac{dT}{T}=0$$

1. Fluid Flow and Thermodynamics

Nozzle fluid flow

- governing equations for analysis of nozzle

$$\frac{d\rho}{\rho}+\frac{du}{u}+\frac{dA}{A}=0$$

$$-M^2\frac{du}{u}+\frac{du}{u}+\frac{dA}{A}=0$$

$$\frac{d\rho}{\rho}=-M^2\frac{du}{u}$$

$$-\frac{dp}{\rho}=udu$$

$$\frac{dA}{A}=\left(M^2-1\right)\frac{du}{u}$$

$$dh+udu=0$$

$$\frac{dp}{p}-\frac{d\rho}{\rho}-\frac{dT}{T}=0$$

1. Fluid Flow and Thermodynamics

Nozzle fluid flow

- governing equations for analysis of nozzle

$$\frac{dA}{A} = \left(M^2 - 1\right)\frac{du}{u}$$

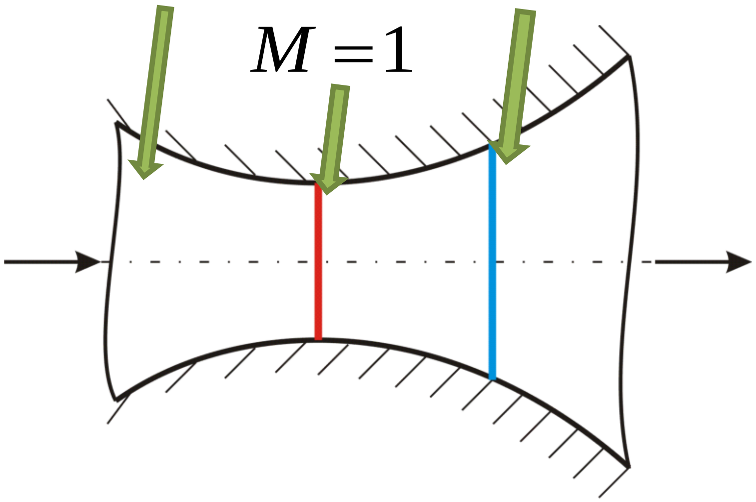
1. subsonic flow: $0 \le M < 1$ → increase in velocity $+du$ is associated with decrease in area $-dA$
2. supersonic flow: $M > 1$ → increase in velocity $+du$ is associated with increase in area $+dA$
3. sonic flow: $M = 1$ → $dA = 0$ area reaches an extremum – minimum

- governing equations for analysis of nozzle

$$\frac{dA}{A} = \left(M^2 - 1\right)\frac{du}{u}$$

$0 \le M < 1$ $\quad M > 1$

$M = 1$



increase in velocity $+du$ is associated with decrease in area $-dA$

increase in velocity $+du$ is associated with increase in area $+dA$

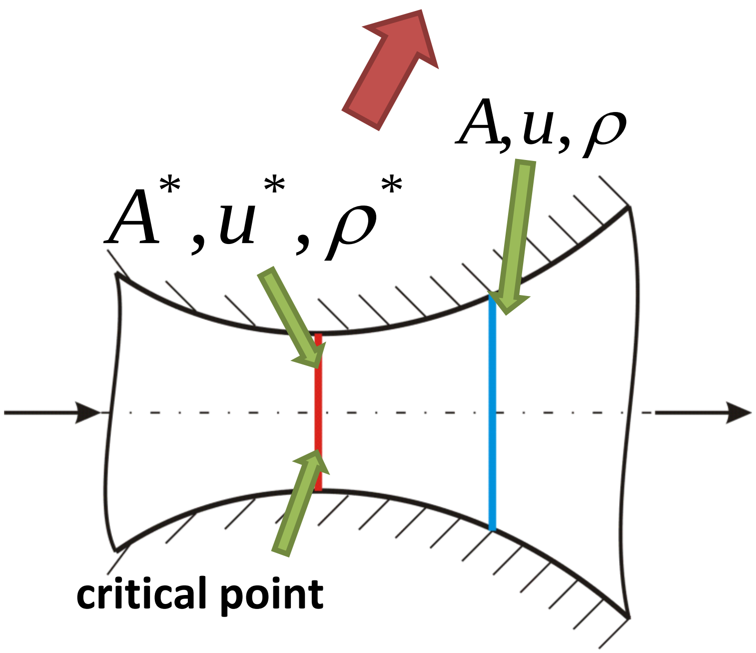
$dA = 0$ area reaches an extremum – minimum

1. Fluid Flow and Thermodynamics

Nozzle fluid flow

- governing equations for analysis of nozzle

$$A^* u^* \rho^* = A u \rho$$



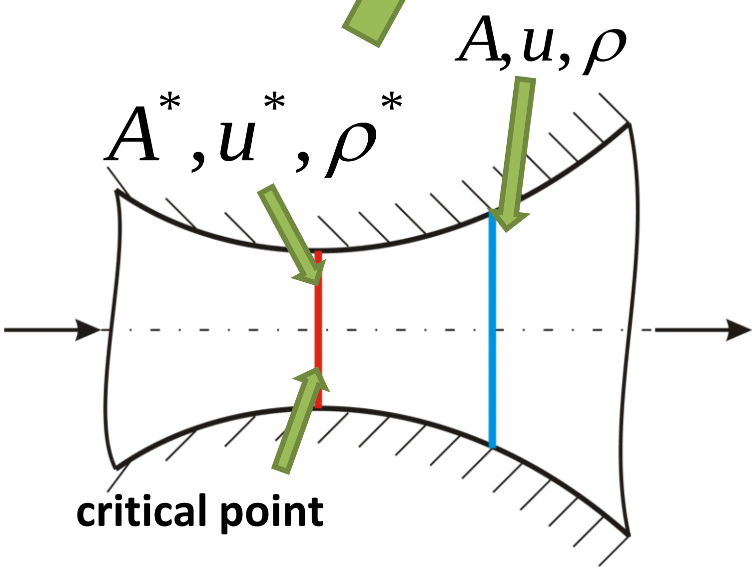
1. Fluid Flow and Thermodynamics

Nozzle fluid flow

- governing equations for analysis of nozzle

$$\frac{A}{A^*} = \frac{u^*}{u}\frac{\rho^*}{\rho} = \frac{a^*}{u}\frac{\rho^*}{\rho_0}\frac{\rho_0}{\rho}$$

$$A^* u^* \rho^* = Au\rho$$



1. Fluid Flow and Thermodynamics

Nozzle fluid flow

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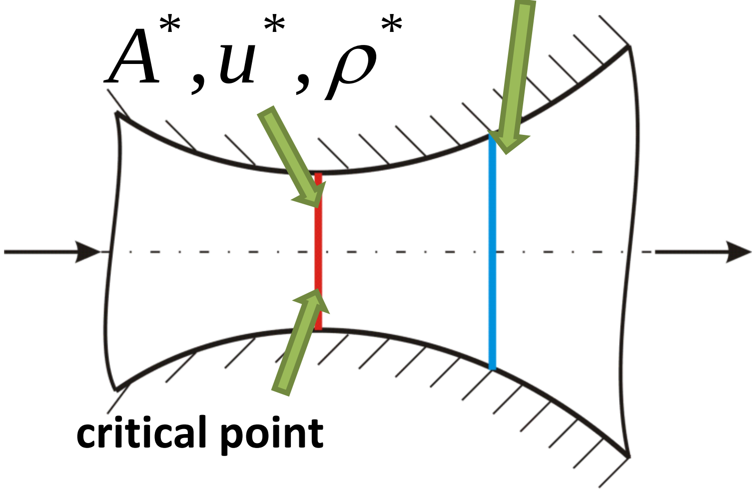


$$\frac{\rho_0}{\rho} = \left(1 + \frac{(\gamma - 1)}{2} M^2\right)^{\frac{1}{\gamma - 1}}$$

$$\frac{\rho_0}{\rho^*} = \left(1 + \frac{(\gamma - 1)}{2}\right)^{\frac{1}{\gamma - 1}}$$

$$\left(M^*\right)^2 = \left(\frac{u}{a^*}\right)^2 = \frac{(\gamma + 1) M^2}{2 + (\gamma - 1) M^2}$$

1. Fluid Flow and Thermodynamics

Nozzle fluid flow

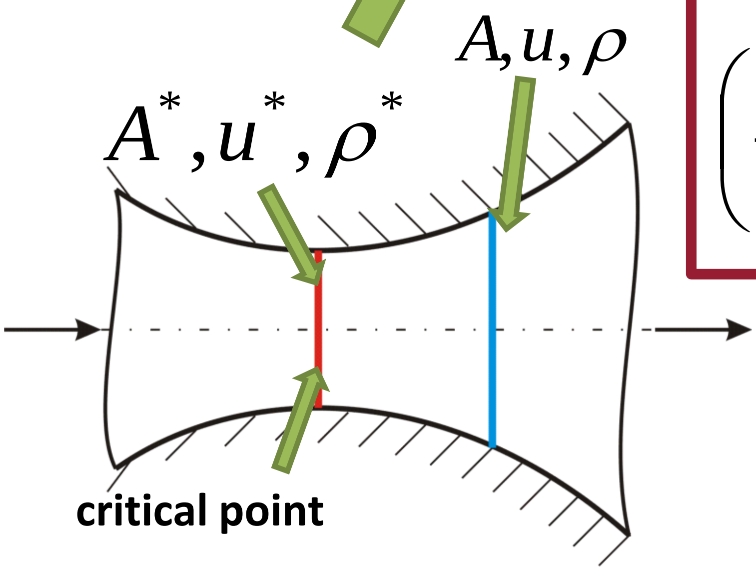
- governing equations for analysis of nozzle

$$A^* u^* \rho^* = A u \rho$$

$$\frac{A}{A^*} = \frac{u^*}{u}\frac{\rho^*}{\rho} = \frac{a^*}{u}\frac{\rho^*}{\rho_0}\frac{\rho_0}{\rho}$$

$$\left(\frac{A}{A^*}\right)^2 = \frac{1}{M^2}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M^2\right)\right]^{\frac{(\gamma+1)}{(\gamma-1)}}$$

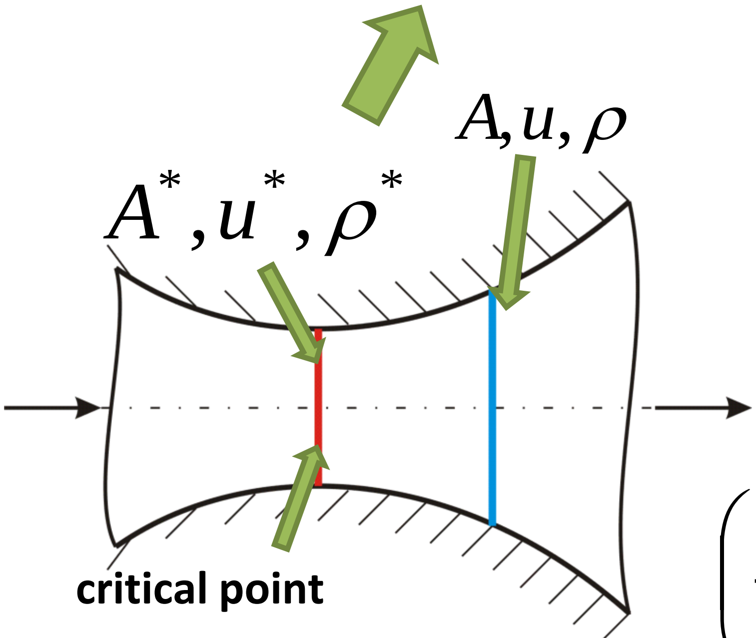
area – Mach number relation

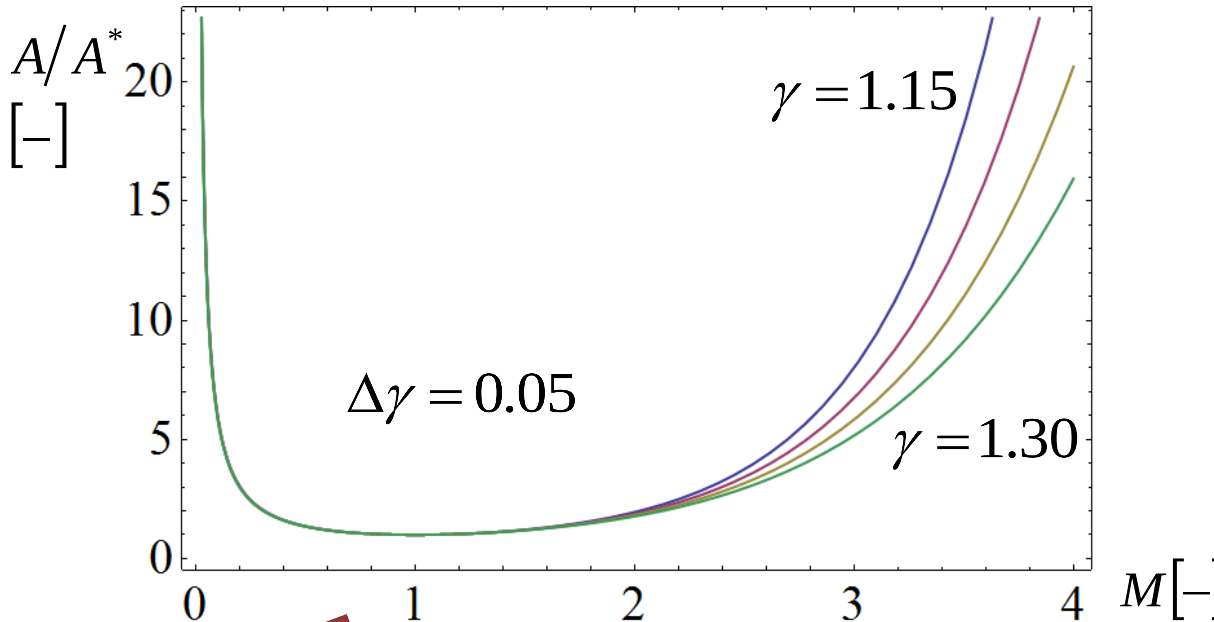


critical point

- governing equations for analysis of nozzle

$$A^* u^* \rho^* = A u \rho$$





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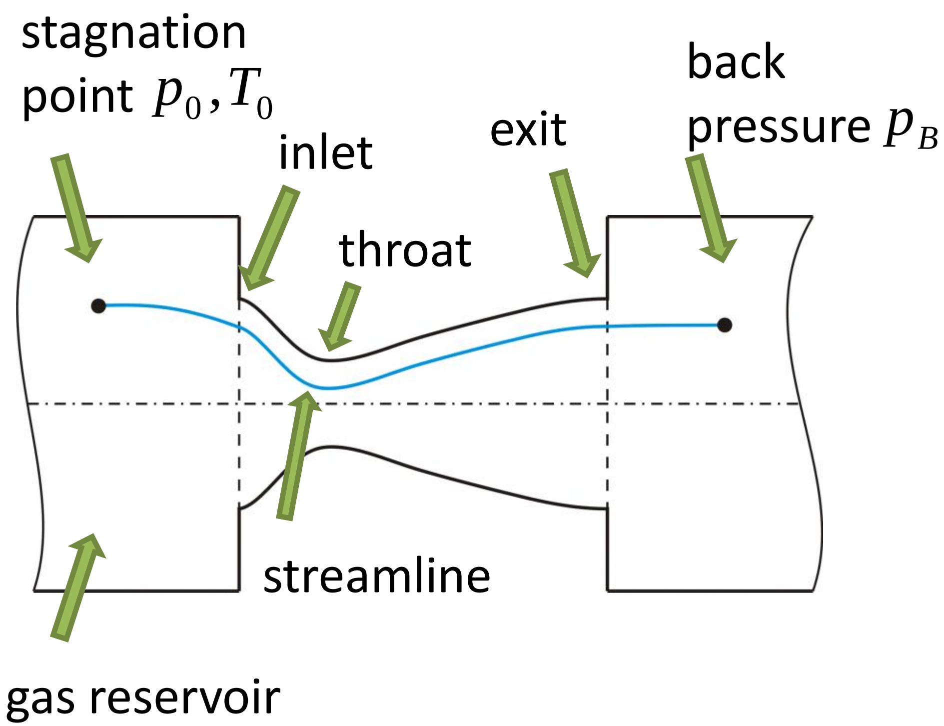
$$\left(\frac{A}{A^*}\right)^2 = \frac{1}{M^2}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M^2\right)\right]^{\frac{(\gamma+1)}{(\gamma-1)}}$$

1. Fluid Flow and Thermodynamics

Nozzle fluid flow

- variation of parameters in nozzle



- stagnation parameters:

$$T_0 = 500\,°\mathrm{K}$$
$$p_0 = 1\mathrm{MPa}$$

- nozzle area:
  inlet area (location 0 m): 0.004 m^2
  throat area (loc. 0.05 m): 0.002 m^2
  exit area (loc. 0.2 m): 0.004 m^2

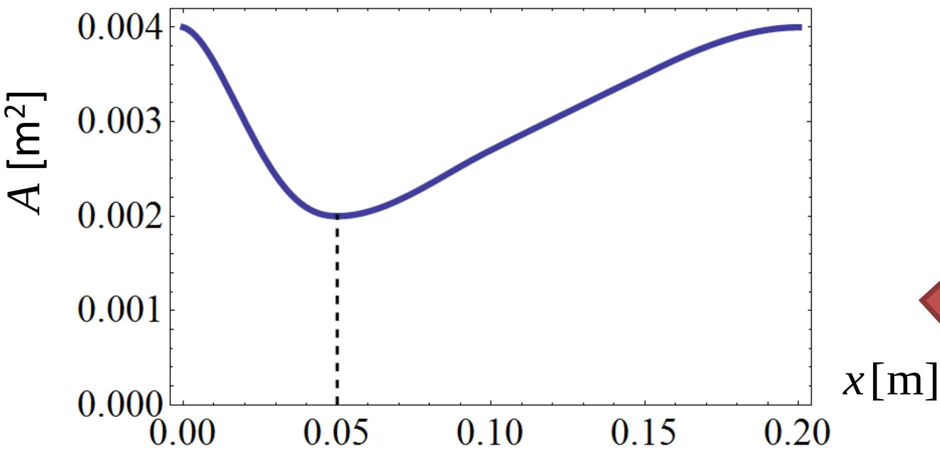
- air:

$$R = 288\ \mathrm{J/kgK}$$
$$\gamma = 1.4$$

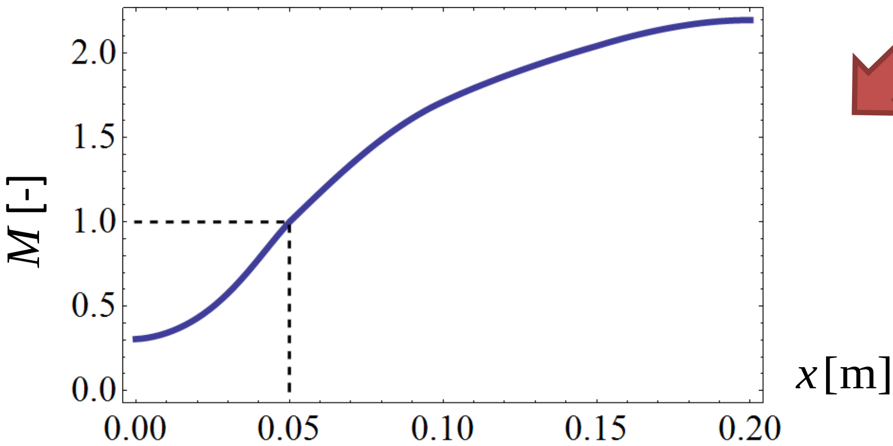
1. Fluid Flow and Thermodynamics

Nozzle fluid flow

- variation of parameters in nozzle



$$\left(\frac{A}{A^*}\right)^2 = \frac{1}{M^2}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M^2\right)\right]^{\frac{(\gamma+1)}{(\gamma-1)}}$$



1. Fluid Flow and Thermodynamics

Nozzle fluid flow

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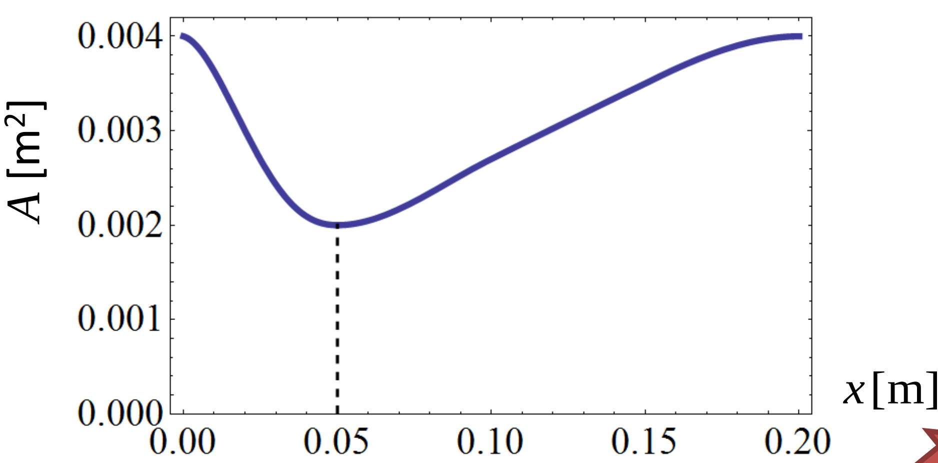
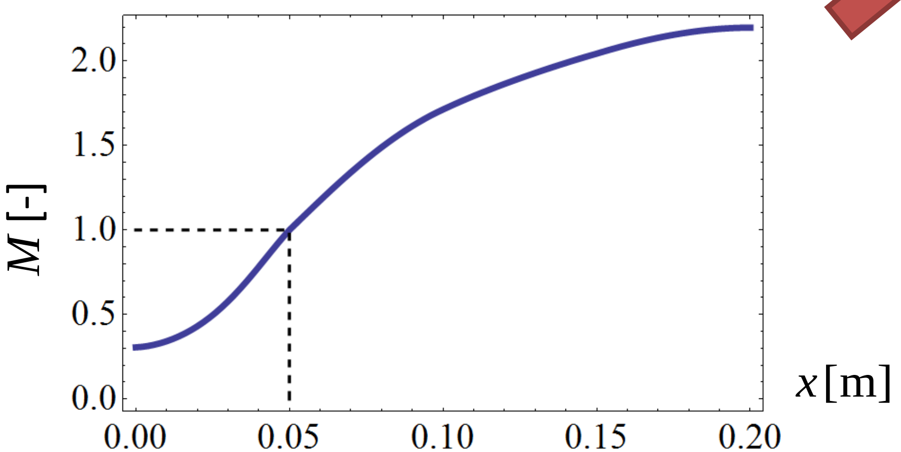
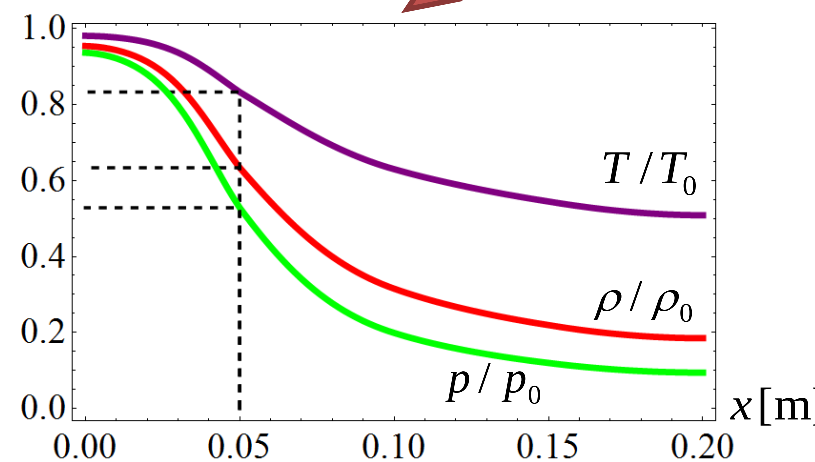


$$1+\frac{(\gamma-1)}{2}M^2=\frac{T_0}{T}$$

$$\left(1+\frac{(\gamma-1)}{2}M^2\right)^{\frac{1}{\gamma-1}}=\frac{\rho_0}{\rho}$$

$$\left(1+\frac{(\gamma-1)}{2}M^2\right)^{\frac{\gamma}{\gamma-1}}=\frac{p_0}{p}$$

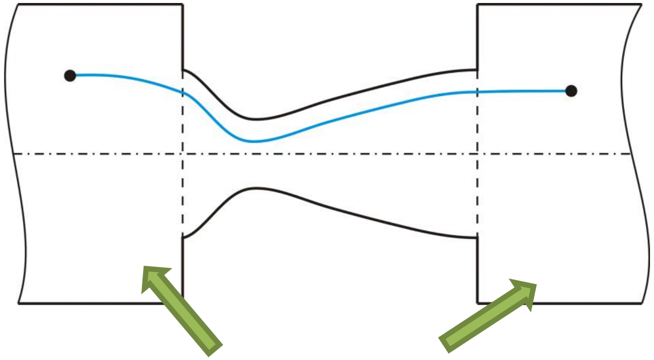




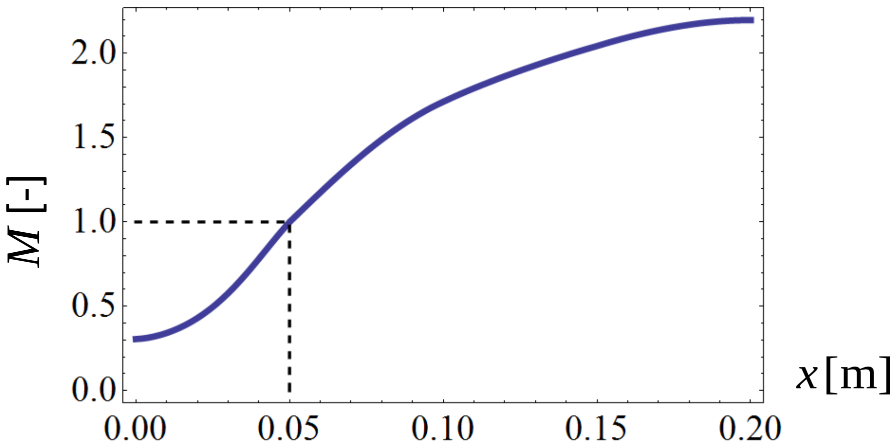
1. Fluid Flow and Thermodynamics

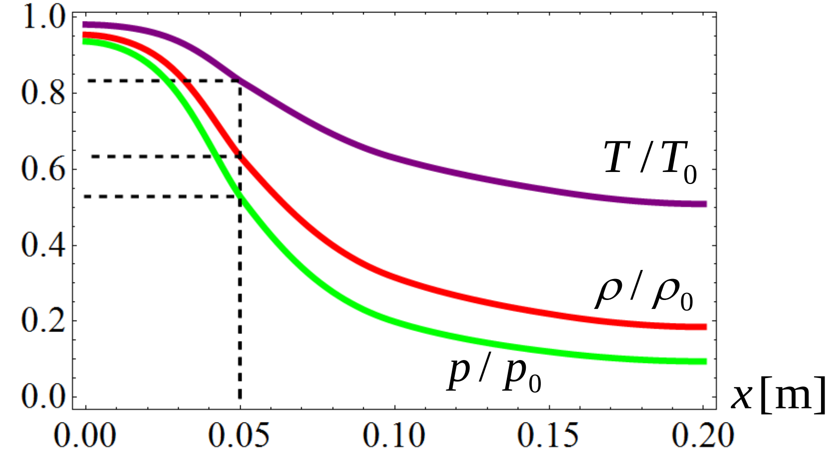
Nozzle fluid flow

- variation of parameters in nozzle
  - no pressure difference – no fluid flow
  - if pressure ratio p_e/p_0 is different from isentropic value, the flow will be different (inside or outside the nozzle)
  - exit pressure for isentropic flow with supersonic speed is p_e



pressure difference causes fluid flow

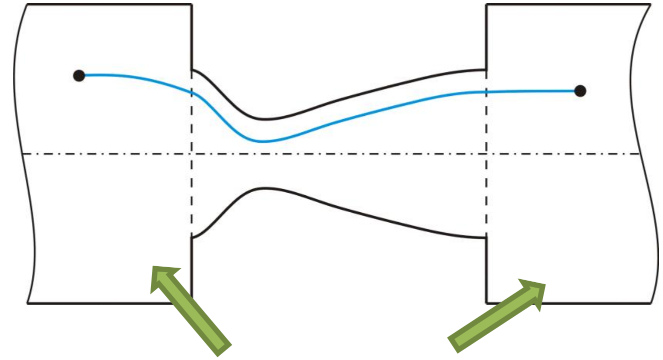




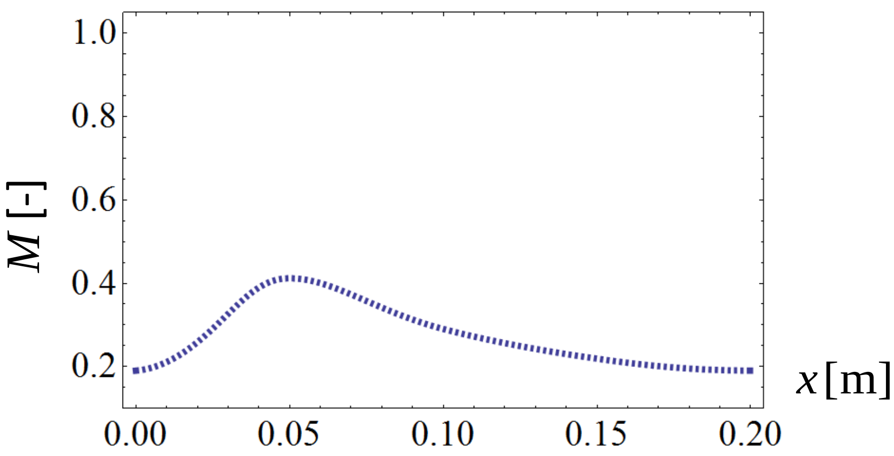
1. Fluid Flow and Thermodynamics

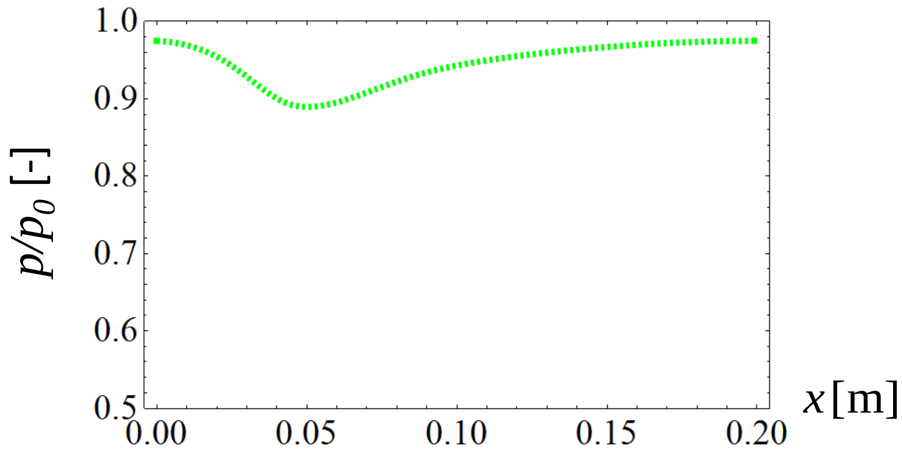
- variation of parameters in nozzle
  - $p_{B,1}$ is reduce below p_0

very low-speed subsonic flow, $p_{B,1} = p_{e,1}$



pressure difference causes fluid flow



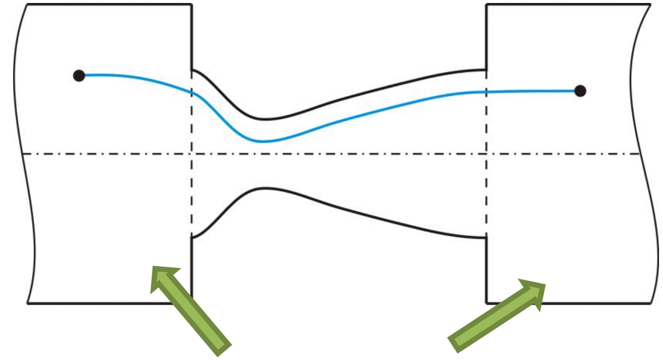


1. Fluid Flow and Thermodynamics

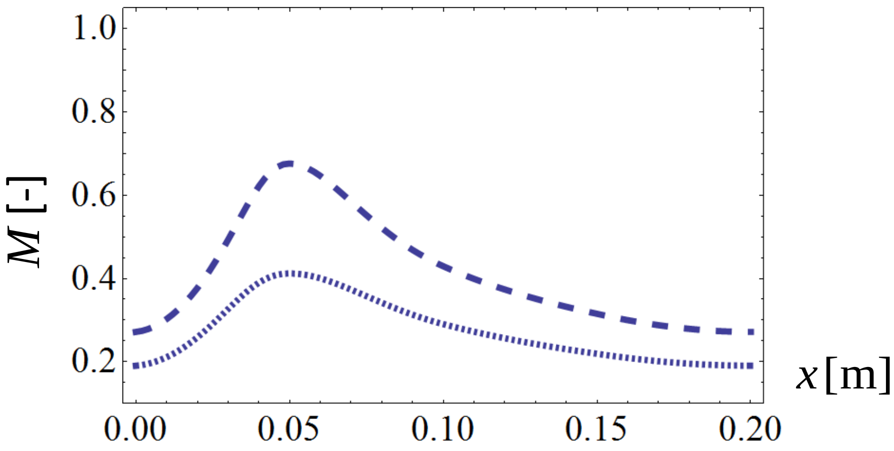
Nozzle fluid flow

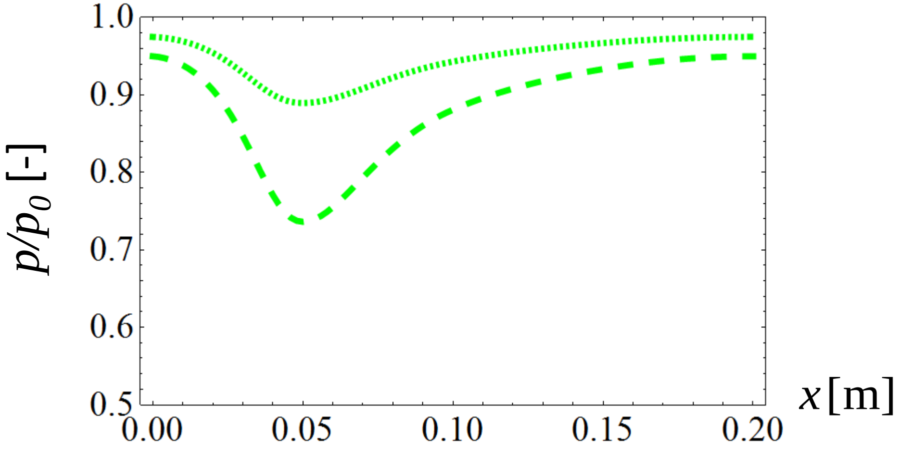
- variation of parameters in nozzle
  - $p_{B,2}$ is reduce below $p_{B,1}$

flow moves faster through nozzle, still subsonic flow, mass flow increases, $p_{B,2} = p_{e,2}$



pressure difference causes fluid flow





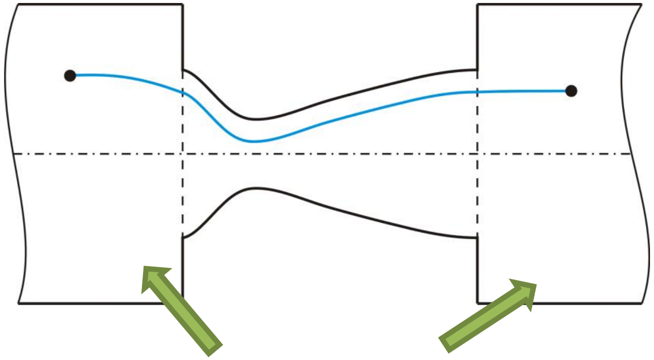
1. Fluid Flow and Thermodynamics

Nozzle fluid flow

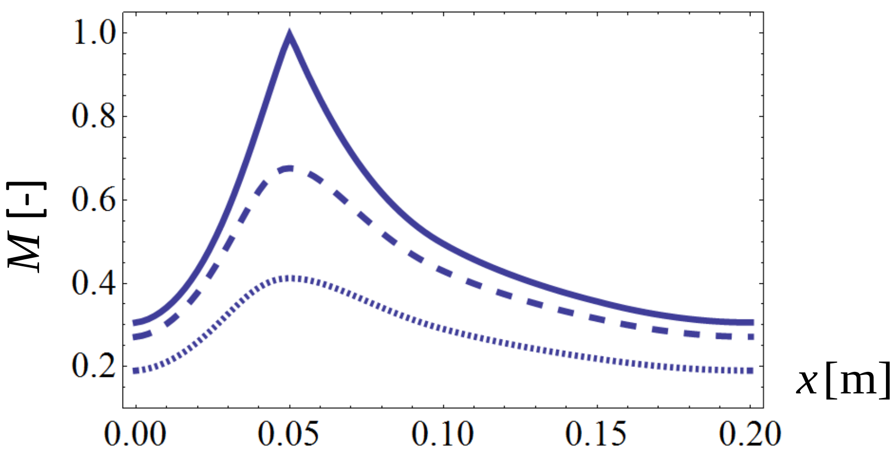
- variation of parameters in nozzle

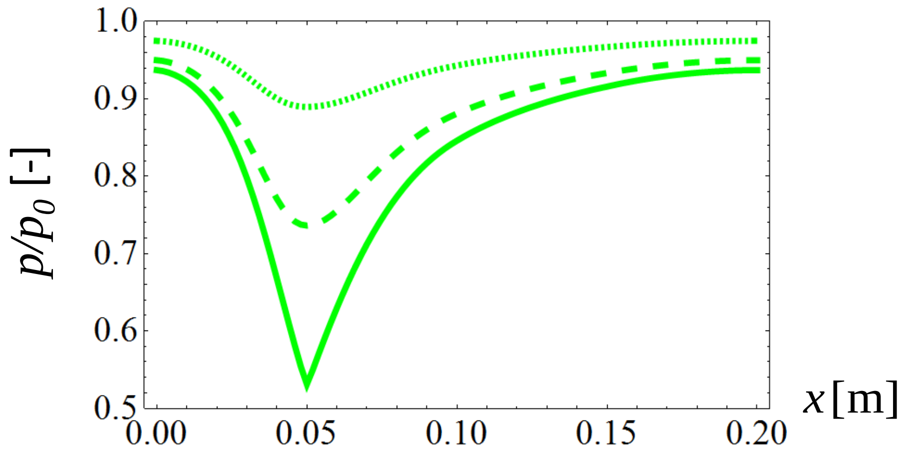
  - $p_{B,3}$ is such, that it produces sonic flow in throat

only in throat is flow sonic, in other parts of nozzle is flow subsonic, mass flow increases and reaches max. value, $p_{B,2} = p_{e,2}$



pressure difference causes fluid flow



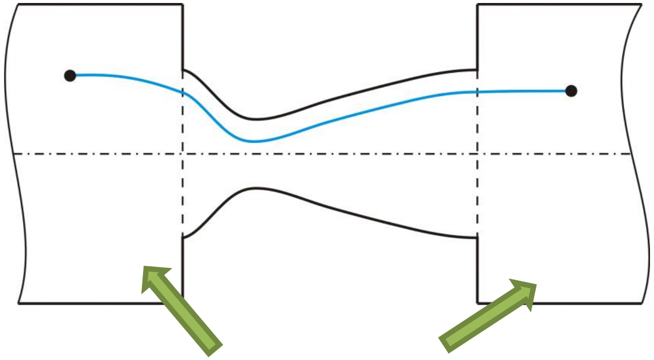


1. Fluid Flow and Thermodynamics

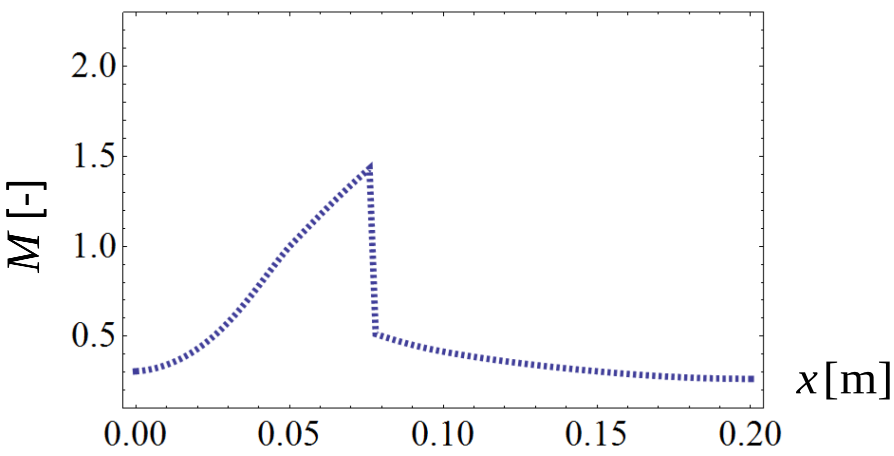
Nozzle fluid flow

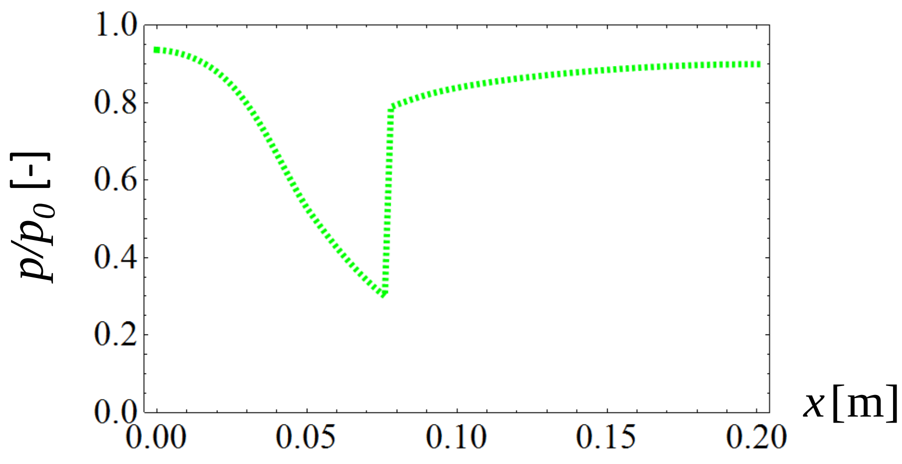
- variation of parameters in nozzle
  - $p_{B,4}$ is reduce below $p_{B,3}$

in divergent nozzle flow is at first supersonic, than shock wave is formed and flow is subsonic, mass flow is constant – chocked flow, $p_{B,4} = p_{e,4}$



pressure difference causes fluid flow



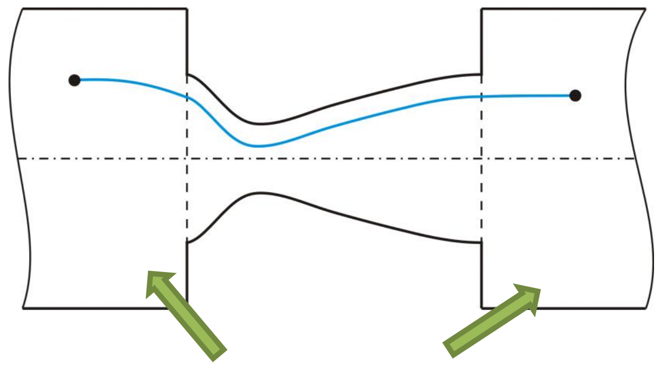


1. Fluid Flow and Thermodynamics

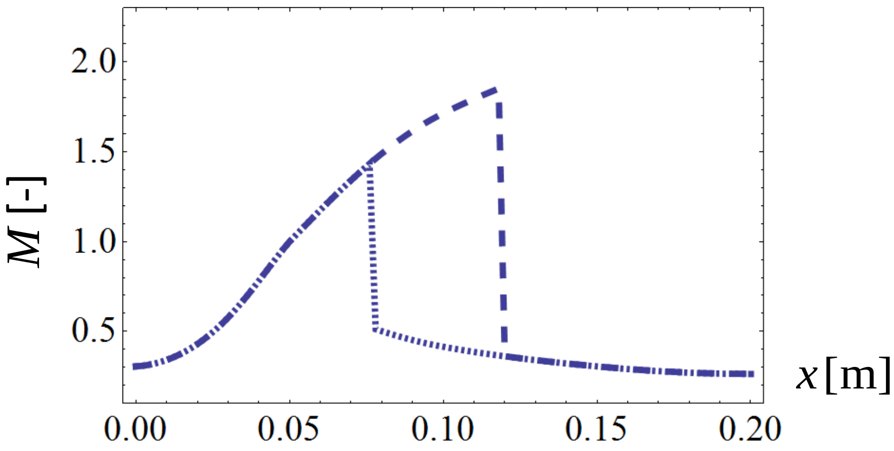
Nozzle fluid flow

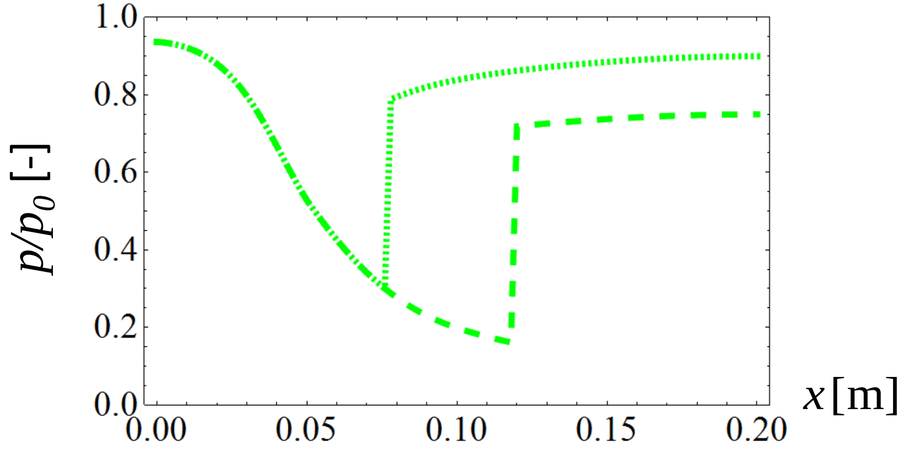
- variation of parameters in nozzle
  - $p_{B,5}$ is reduce below $p_{B,4}$

shock wave is moving toward the exit plane, $p_{B,4} = p_{e,4}$



pressure difference causes fluid flow





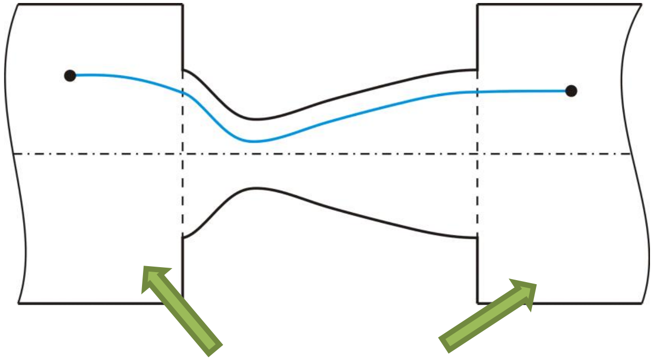
1. Fluid Flow and Thermodynamics

Nozzle fluid flow

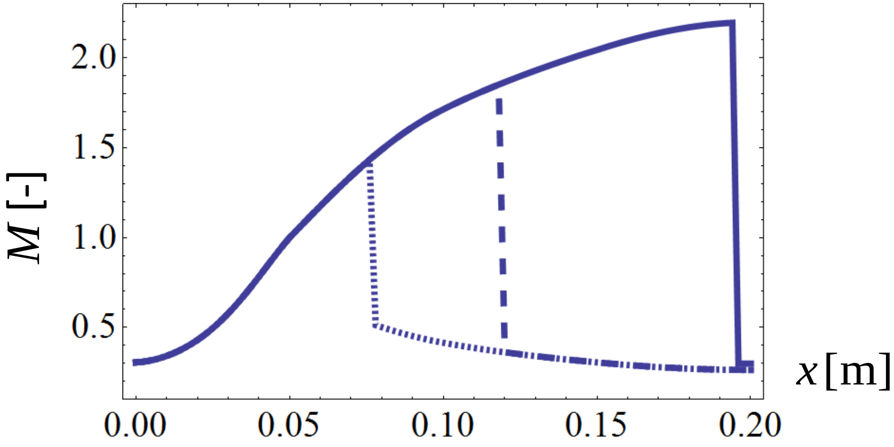
- variation of parameters in nozzle

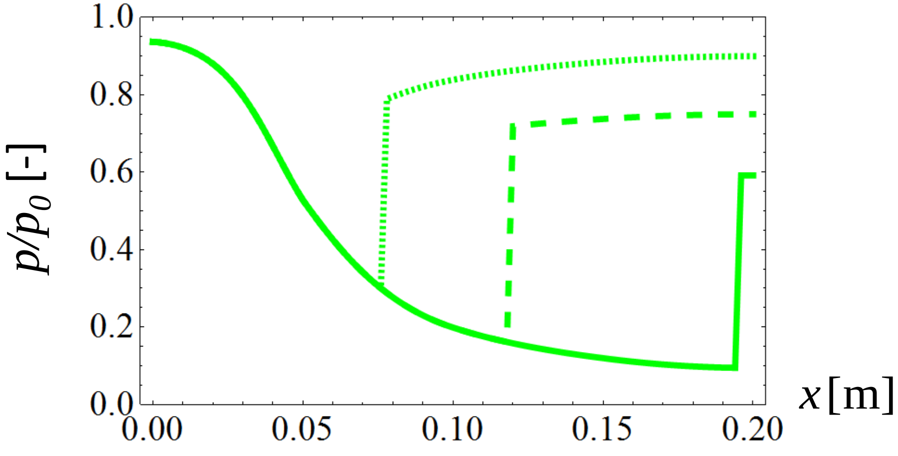
  - $p_{B,6}$ is such, that shock wave is on the exit plane

the flow is supersonic in whole nozzle except the exit plane, $p_{B,6} = p_{e,6}$



pressure difference causes fluid flow





1. Fluid Flow and Thermodynamics

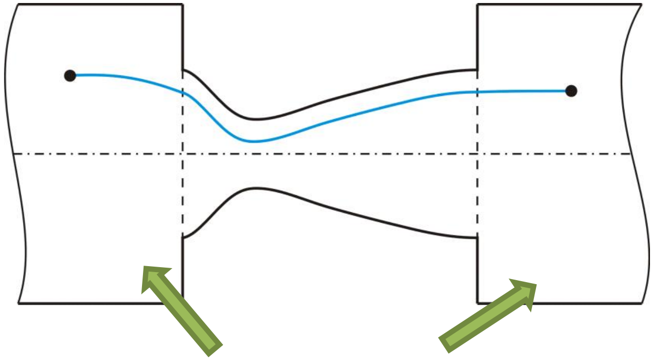
Nozzle fluid flow

- variation of parameters in nozzle

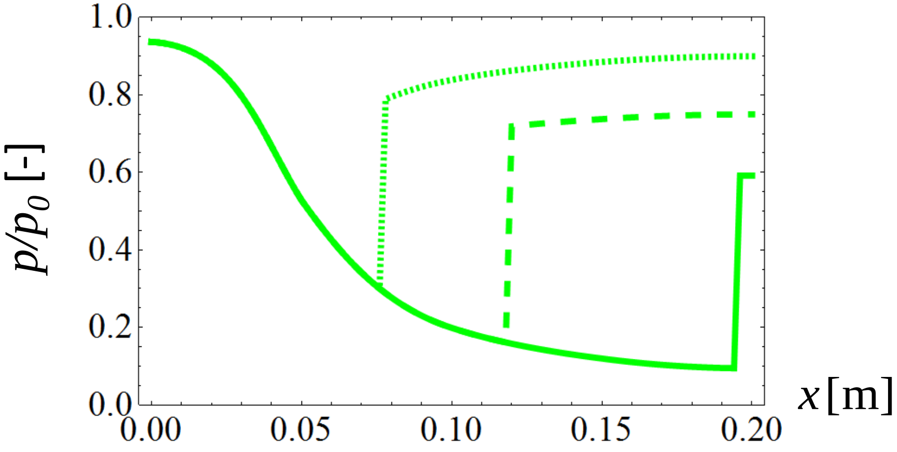
  - $p_{B,6}$ is such, that shock wave is on the exit plane

the flow is supersonic in whole nozzle except the exit plane, $p_{B,6} = p_{e,6}$

other reduction of back pressure p_B:
- exit pressure is constant p_e
- if $p_B > p_e$ shock waves moves outside nozzle
- if $p_B = p_e$ no shock waves are produced
- if $p_B < p_e$ expansion waves are formed outside the nozzle



pressure difference causes fluid flow



1. Fluid Flow and Thermodynamics

Nozzle fluid flow

- variation of parameters in nozzle

  - $p_{B,6}$ is such, that shock wave is on the exit plane

↓

the flow is supersonic in whole nozzle except the exit plane, $p_{B,6} = p_{e,6}$

↓

other reduction of back pressure p_B:
- exit pressure is constant p_e
- if $p_B > p_e$ shock waves moves outside nozzle
- if $p_B = p_e$ no shock waves are produced
- if $p_B < p_e$ expansion waves are formed outside the nozzle

over-expanded flow $p_B > p_e$ (low altitudes)

under-expanded flow $p_B < p_e$ (high altitudes)

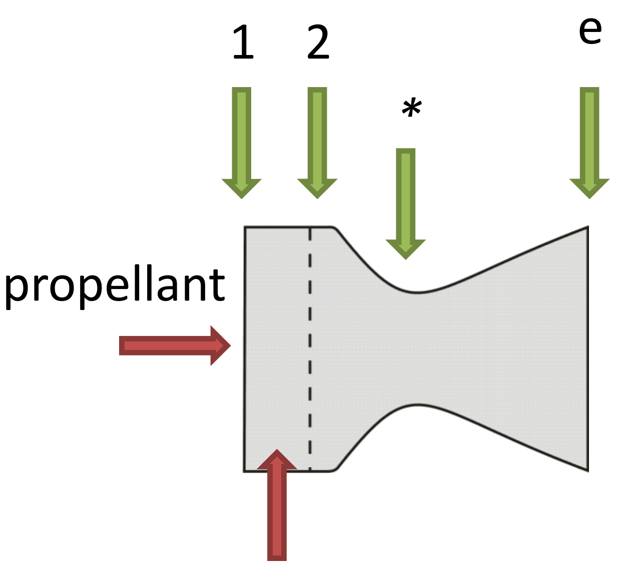
source: NASA

2. Chemical Rocket Propulsion

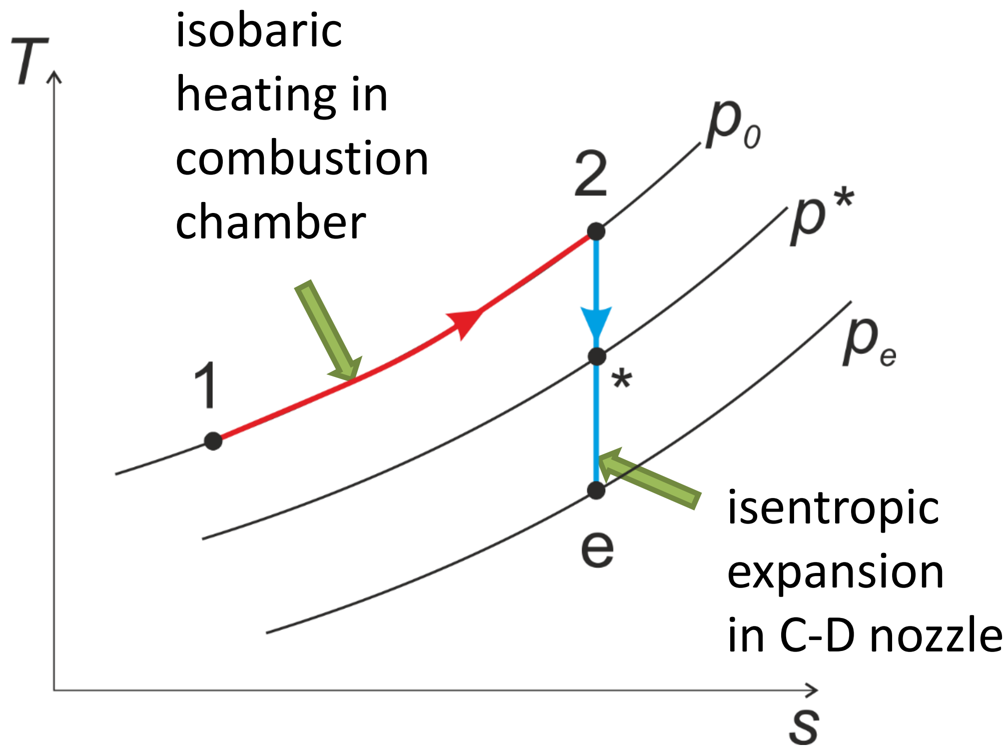
- Performance Characteristics
- Liquid Propellant Performance
- Feed System

2. Chemical Rocket Propulsion

- Rocket thrust chamber – combustion



heat per unit mass q_R

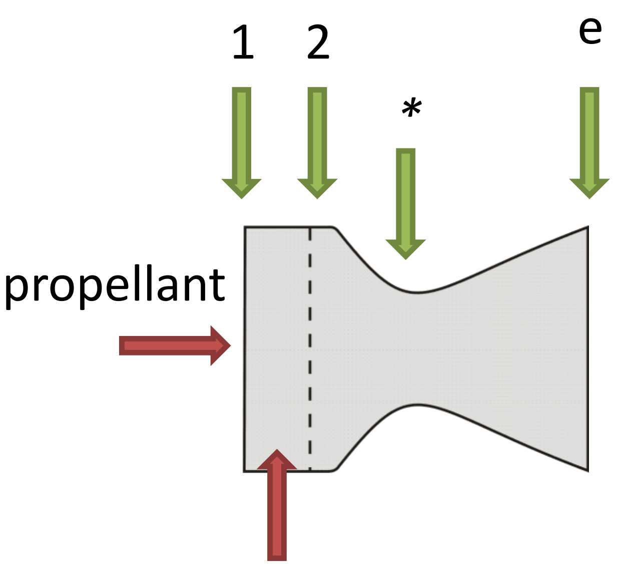


First thermodynamic law – isobaric heating: $q_R = \Delta h = c_p \Delta T$

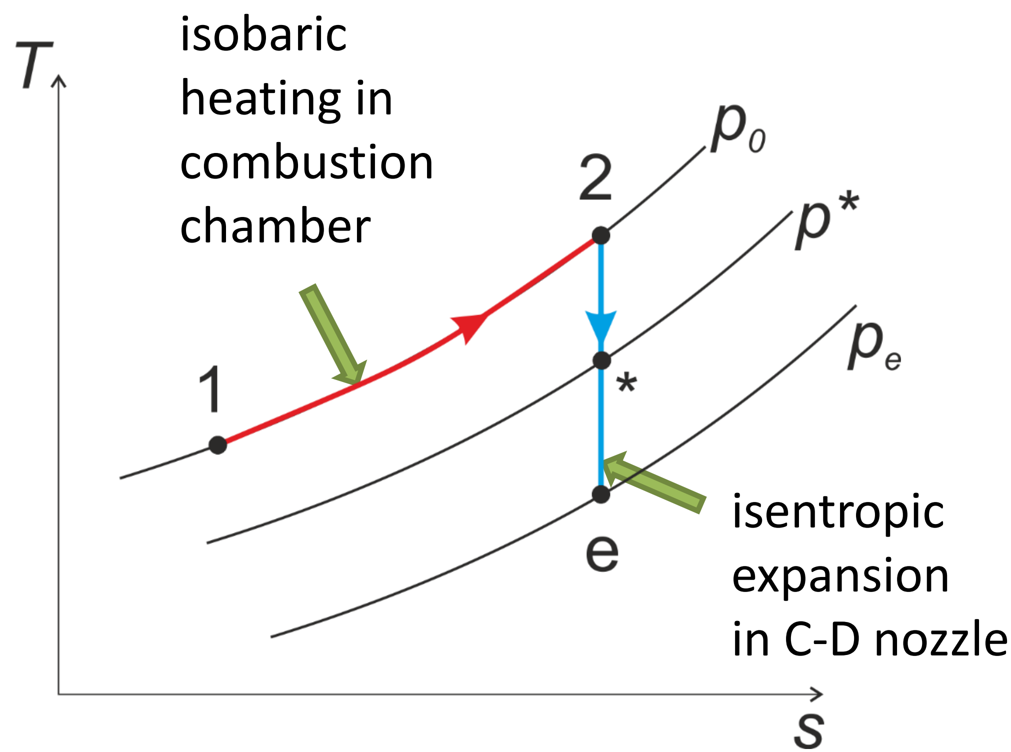
2. Chemical Rocket Propulsion

Performance Characteristics

- Rocket thrust chamber – combustion



heat per unit mass q_R



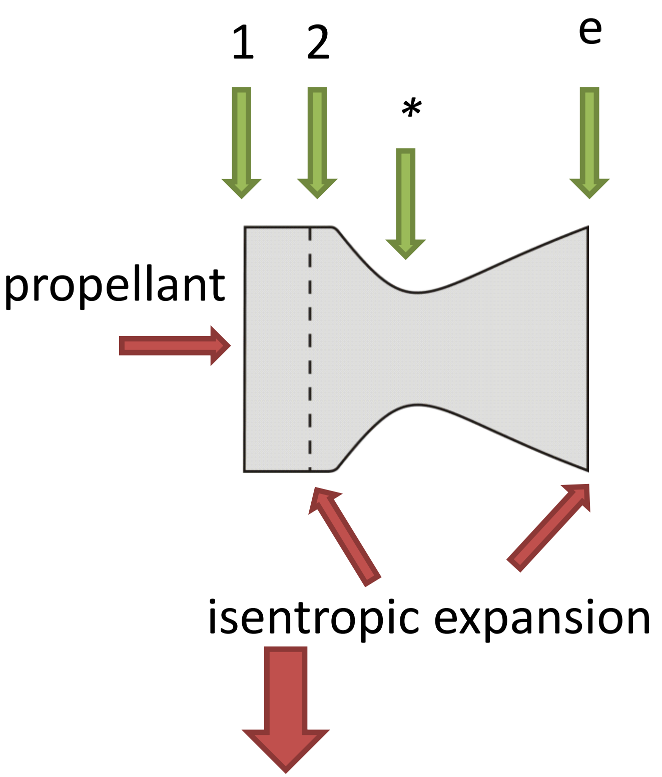
First thermodynamic law – isobaric heating: $q_R = \Delta h = c_p \Delta T$ ⇒

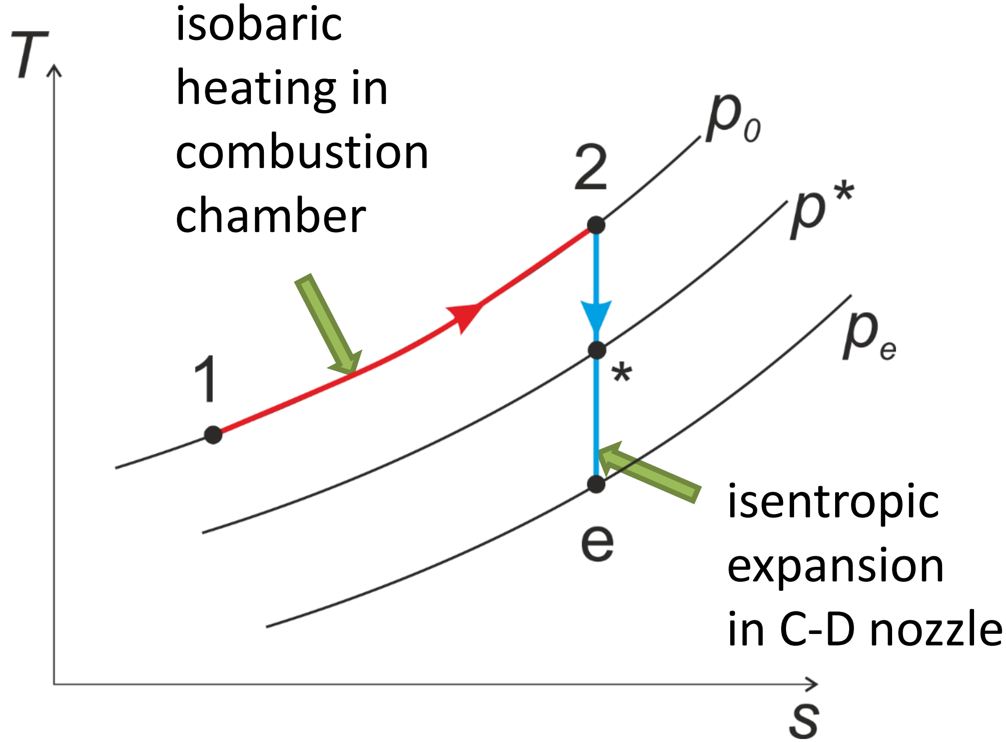
$$T_{02} = T_{01} + \frac{q_R}{c_p}$$

2. Chemical Rocket Propulsion

Performance Characteristics

- Rocket thrust chamber – expansion

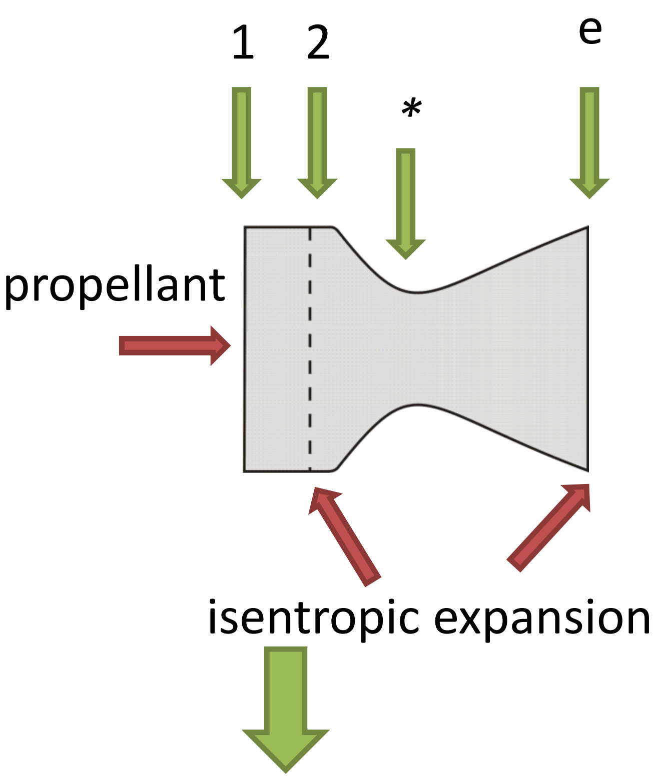


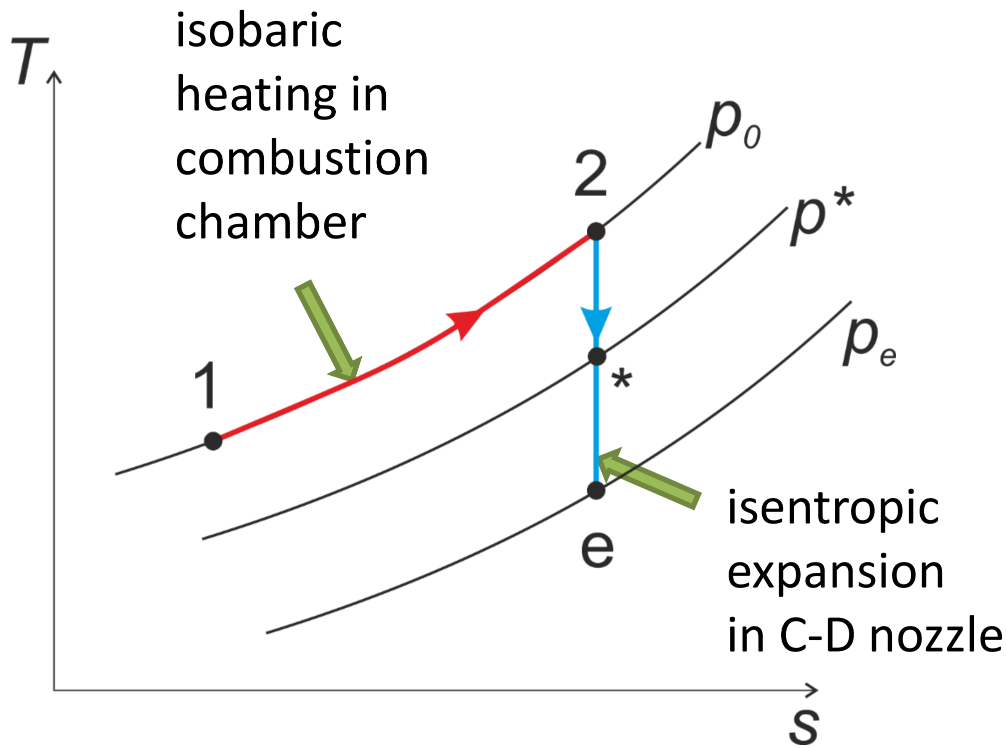


stagnation enthalpy: $h_{02} = h_{0e} \Rightarrow h_{02} = \frac{u_e^2}{2} + h_e$

2. Chemical Rocket Propulsion

- Rocket thrust chamber – expansion



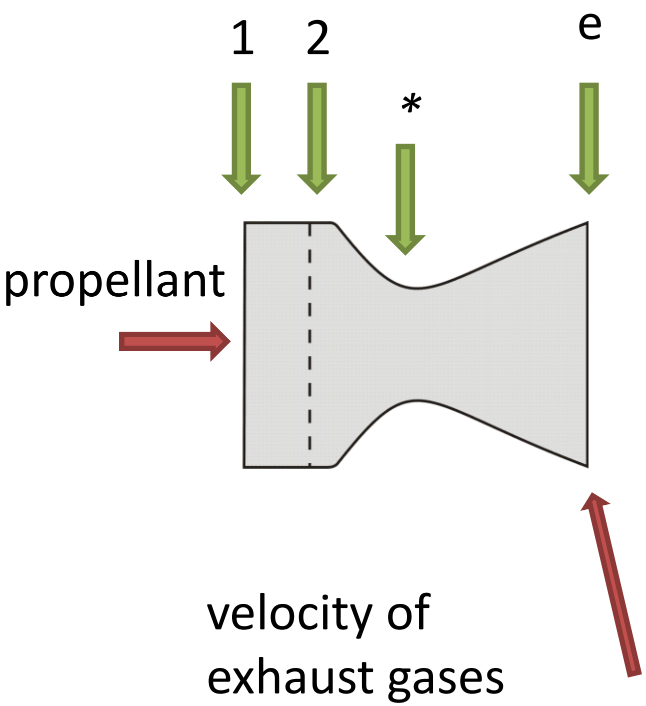


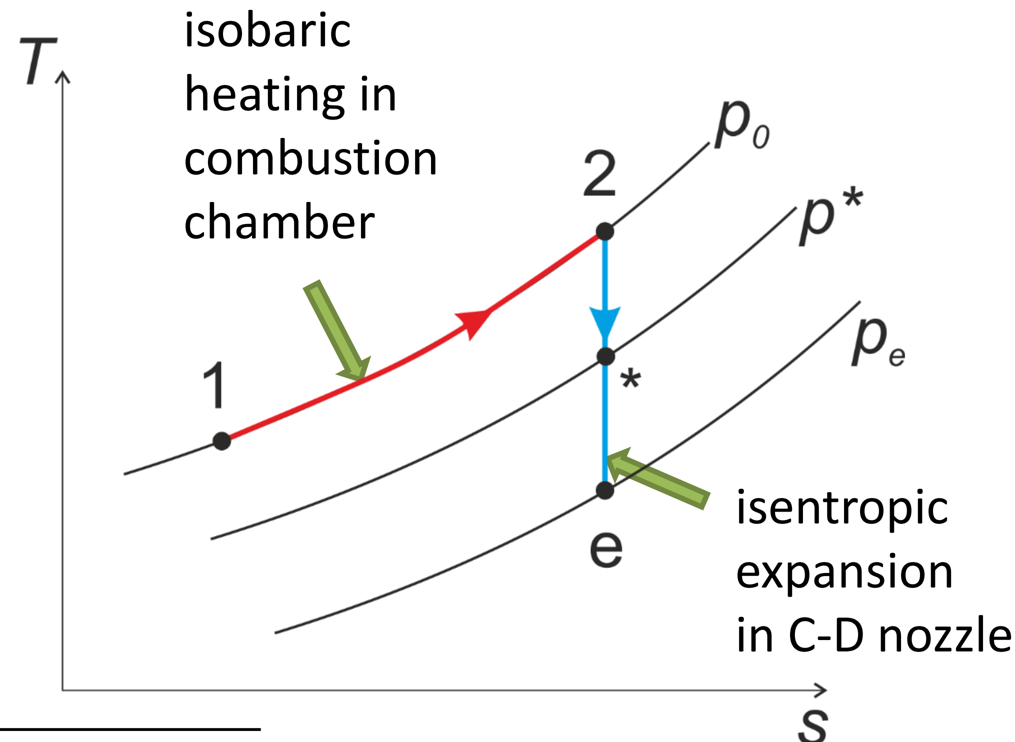
stagnation enthalpy: $h_{02} = h_{0e}$ ⇒ $h_{02} = \frac{u_e^2}{2} + h_e$ ⇒ $u_e = \sqrt{2\left(h_{02} - h_e\right)} = \sqrt{2c_p\left(T_{02} - T_e\right)}$

2. Chemical Rocket Propulsion

Performance Characteristics

- Rocket thrust chamber – expansion



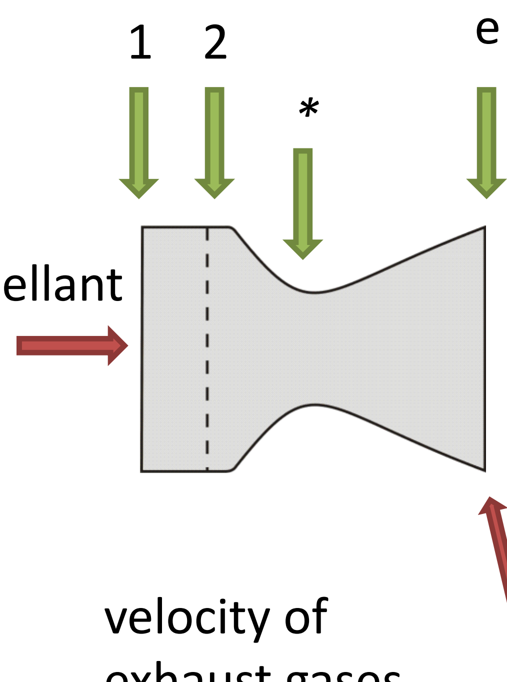


$$u_e = \sqrt{2c_p T_{02}\left(1 - \frac{T_e}{T_{02}}\right)} = \sqrt{2c_p T_{02}\left(1 - \left(\frac{p_e}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right)}$$

$$u_e = \sqrt{2\left(h_{02} - h_e\right)} = \sqrt{2c_p\left(T_{02} - T_e\right)}$$

2. Chemical Rocket Propulsion

- Rocket thrust chamber – exit velocity



$$u_e = \sqrt{2\frac{\gamma}{\gamma-1}RT_{02}\left(1-\left(\frac{p_e}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right)}$$

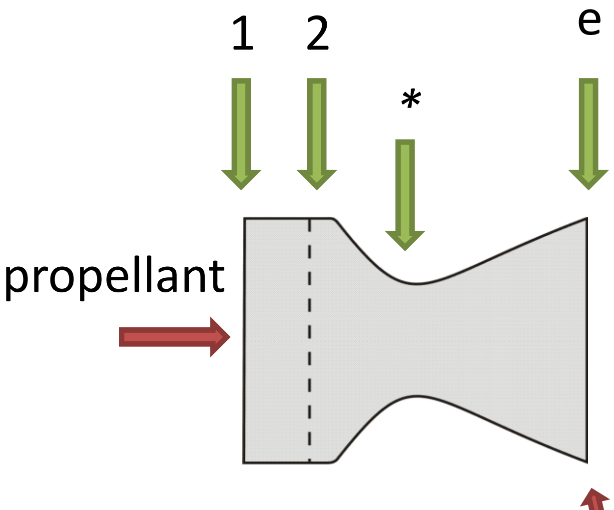
$$c_p = \frac{\gamma R}{\gamma-1}$$

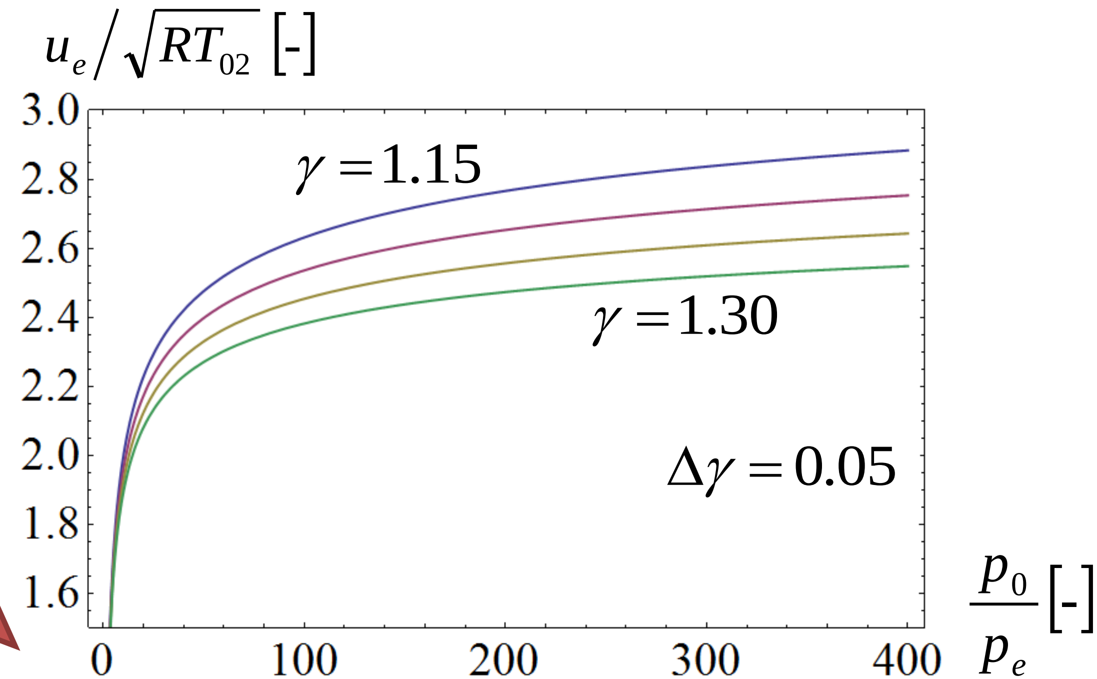
$$u_e = \sqrt{2\left(h_{02}-h_e\right)} = \sqrt{2c_p\left(T_{02}-T_e\right)}$$

$$u_e = \sqrt{2c_pT_{02}\left(1-\frac{T_e}{T_{02}}\right)} = \sqrt{2c_pT_{02}\left(1-\left(\frac{p_e}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right)}$$

2. Chemical Rocket Propulsion

- Rocket thrust chamber – exit velocity





velocity of exhaust gases

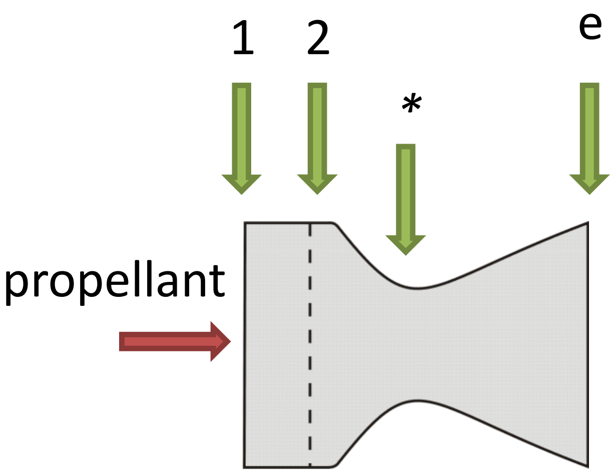
$$u_e = \sqrt{2\frac{\gamma}{\gamma-1}RT_{02}\left(1-\left(\frac{p_e}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right)}$$

$$u_e = \sqrt{2\left(h_{02}-h_e\right)} = \sqrt{2c_p\left(T_{02}-T_e\right)}$$

2. Chemical Rocket Propulsion

Performance Characteristics

- Rocket thrust chamber – exit velocity



$$u_e = \sqrt{2\frac{\gamma}{\gamma-1}\frac{R_M}{M_m}T_{02}\left(1-\left(\frac{p_e}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right)}$$

$$R = \frac{R_M}{M_m}$$

velocity of exhaust gases

$$u_e = \sqrt{2\frac{\gamma}{\gamma-1}RT_{02}\left(1-\left(\frac{p_e}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right)}$$

$$u_e = \sqrt{2\left(h_{02}-h_e\right)} = $$
$$= \sqrt{2c_p\left(T_{02}-T_e\right)}$$

2. Chemical Rocket Propulsion

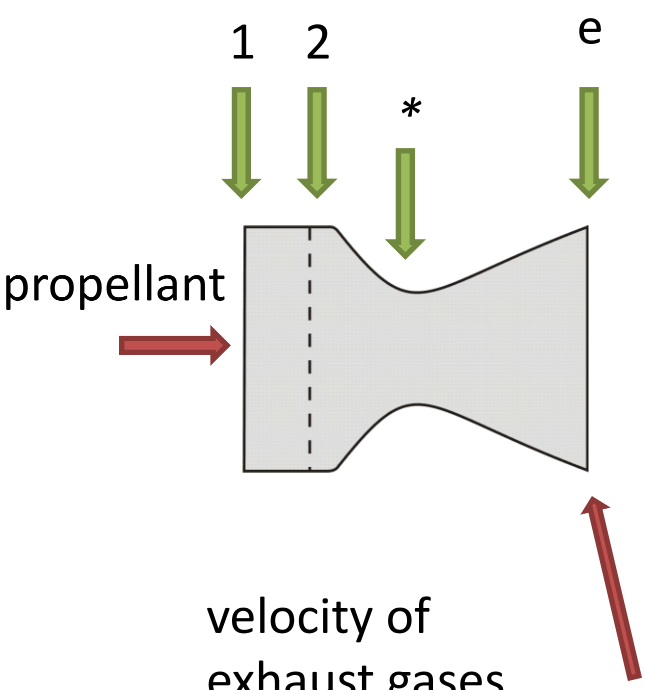
Performance Characteristics

- Rocket thrust chamber – exit velocity

higher stag. temperature → higher exit velocity

low molecular weight → higher exit velocity

higher pressure ratio p_0/p_e → higher exit velocity



$$u_e = \sqrt{2\frac{\gamma}{\gamma-1}\frac{R_M}{M_m}T_{02}\left(1-\left(\frac{p_e}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right)}$$

$$R = \frac{R_M}{M_m}$$

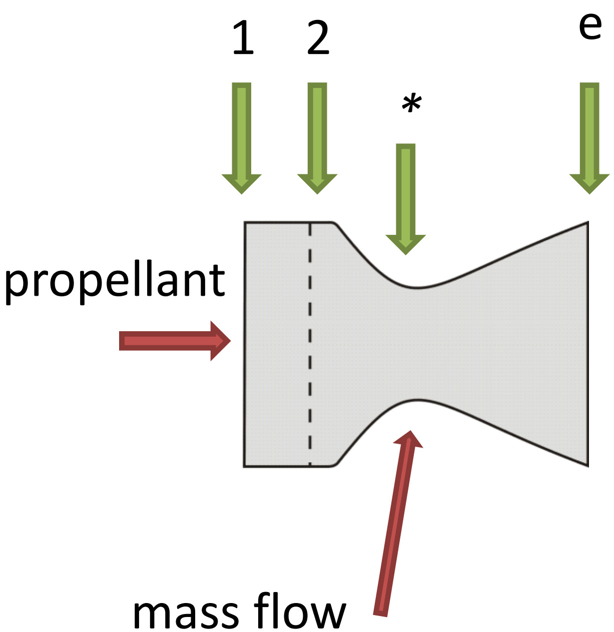
$$u_e = \sqrt{2\left(h_{02}-h_e\right)} = $$

$$= \sqrt{2c_p\left(T_{02}-T_e\right)}$$

$$u_e = \sqrt{2\frac{\gamma}{\gamma-1}RT_{02}\left(1-\left(\frac{p_e}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right)}$$

2. Chemical Rocket Propulsion

- Rocket thrust chamber – mass flow

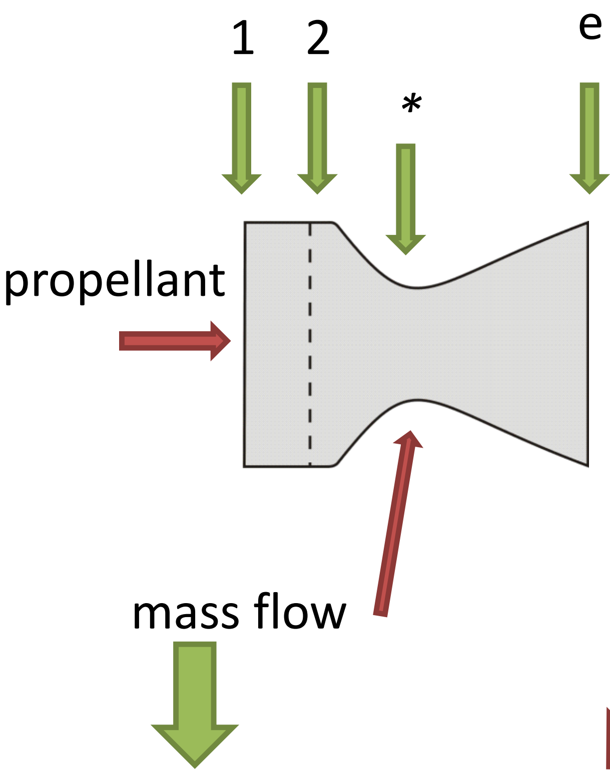


continuity equation: $\dot{m} = A^{*} \rho^{*} u^{*}$

2. Chemical Rocket Propulsion

Performance Characteristics

- Rocket thrust chamber – mass flow



$$u^* = \sqrt{\gamma R T^*} \qquad 1 + \frac{(\gamma - 1)}{2} M^2 = \frac{T_{02}}{T}$$

$$u^* = \sqrt{\gamma R T_{02} \frac{2}{1+\gamma}} = \frac{R T_{02} \sqrt{\gamma}}{\sqrt{R T_{02}}} \left(\frac{1+\gamma}{2} \right)^{-\frac{1}{2}}$$

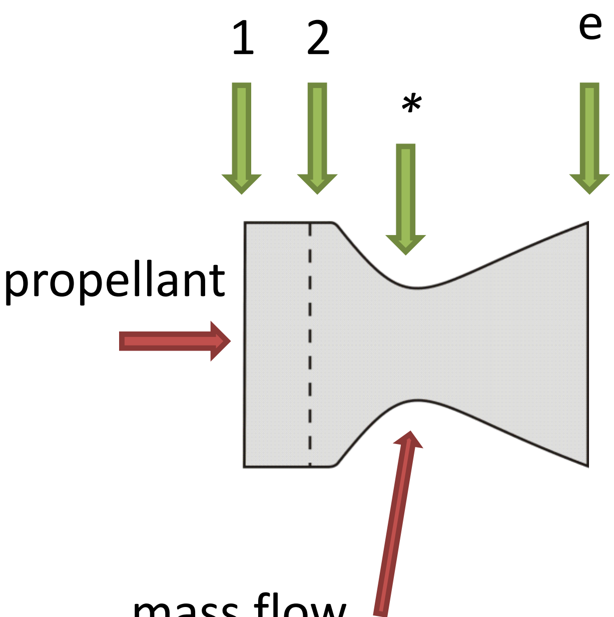
$$\frac{\rho_{02}}{\rho^*} = \left(\frac{1+\gamma}{2} \right)^{\frac{1}{\gamma - 1}}$$

continuity equation: $\dot{m} = A^* \rho^* u^*$

2. Chemical Rocket Propulsion

Performance Characteristics

- Rocket thrust chamber – mass flow



$$u^* = \sqrt{\gamma R T_{02} \frac{2}{1+\gamma}} = \frac{R T_{02}\sqrt{\gamma}}{\sqrt{R T_{02}}}\left(\frac{1+\gamma}{2}\right)^{-\frac{1}{2}}$$

$$\frac{\rho_{02}}{\rho^*} = \left(\frac{1+\gamma}{2}\right)^{\frac{1}{\gamma-1}}$$

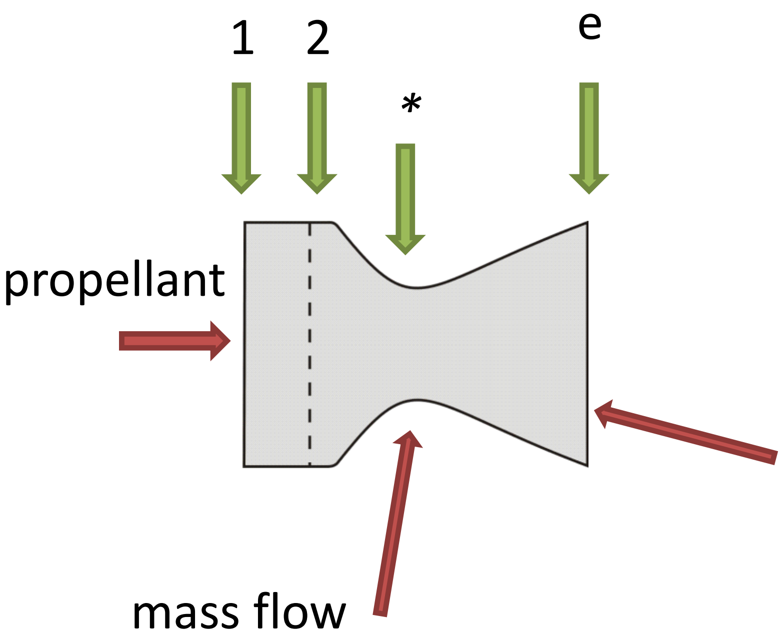
continuity equation: $\dot{m} = A^* \rho^* u^*$

$$\dot{m} = A^* \rho_{02}\left(\frac{1+\gamma}{2}\right)^{\frac{-1}{\gamma-1}} \frac{R T_{02}\sqrt{\gamma}}{\sqrt{R T_{02}}}\left(\frac{1+\gamma}{2}\right)^{-\frac{1}{2}}$$

2. Chemical Rocket Propulsion

Performance Characteristics

- Rocket thrust chamber – mass flow

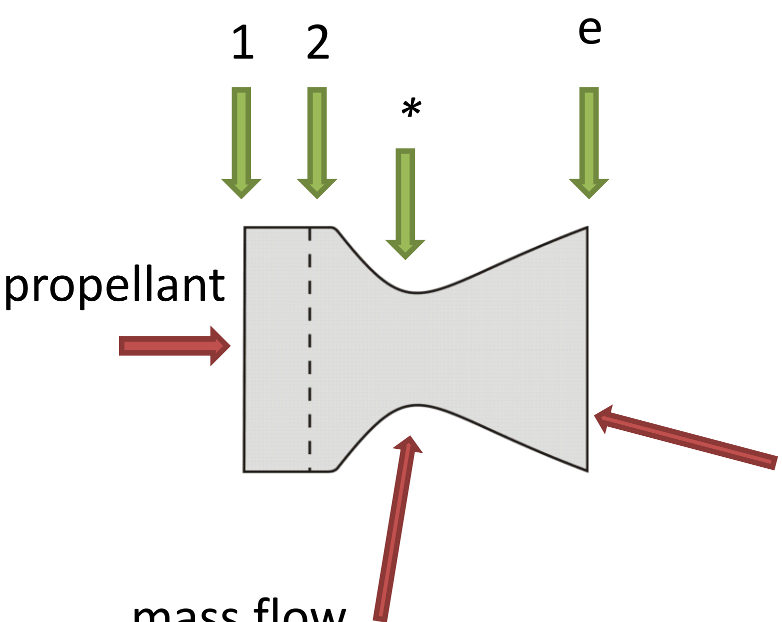


$$\dot{m} = \sqrt{\gamma\left(\frac{2}{1+\gamma}\right)^{\frac{1+\gamma}{(\gamma-1)}}}\;\frac{A^{*} p_0}{\sqrt{RT_{02}}}$$

continuity equation: $\dot{m} = A^{*}\rho^{*}u^{*}$ ⇒ $\dot{m} = A^{*}\rho_{02}\left(\frac{1+\gamma}{2}\right)^{\frac{-1}{\gamma-1}}\frac{RT_{02}\sqrt{\gamma}}{\sqrt{RT_{02}}}\left(\frac{1+\gamma}{2}\right)^{-\frac{1}{2}}$

2. Chemical Rocket Propulsion

- Rocket thrust chamber – mass flow



higher stag. pressure → higher mass flow

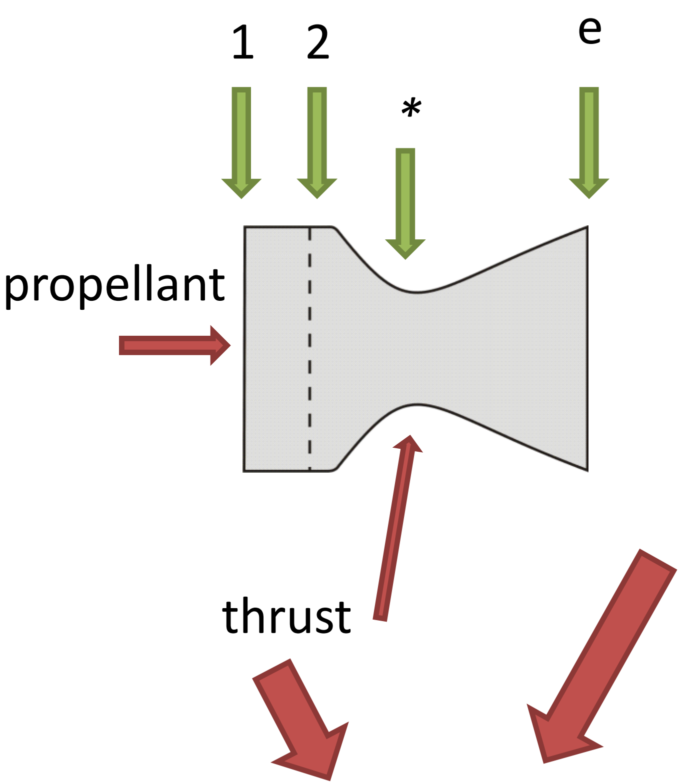
$$\dot{m} = \sqrt{\gamma\left(\frac{2}{1+\gamma}\right)^{\frac{1+\gamma}{(\gamma-1)}}}\,\frac{A^* p_0}{\sqrt{RT_{02}}}$$

continuity equation: $\dot{m} = A^* \rho^* u^*$ → $\dot{m} = A^* \rho_{02}\left(\frac{1+\gamma}{2}\right)^{\frac{-1}{\gamma-1}} \frac{RT_{02}\sqrt{\gamma}}{\sqrt{RT_{02}}}\left(\frac{1+\gamma}{2}\right)^{-\frac{1}{2}}$

2. Chemical Rocket Propulsion

Performance Characteristics

- Rocket thrust chamber – thrust



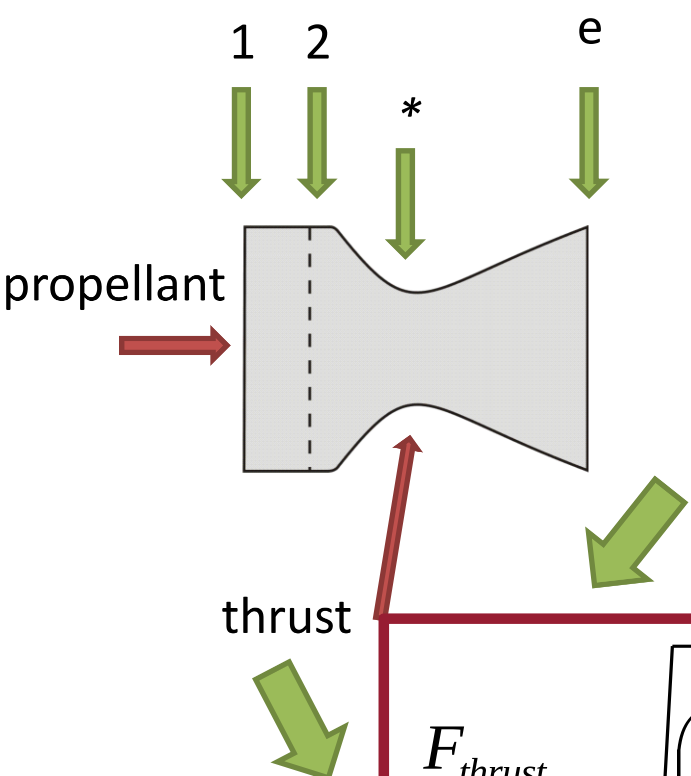
$$u_e = \sqrt{2\frac{\gamma}{\gamma-1}RT_{02}\left(1-\left(\frac{p_e}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right)}$$

$$\dot{m} = \sqrt{\gamma\left(\frac{2}{1+\gamma}\right)^{\frac{1+\gamma}{(\gamma-1)}}}\frac{A^* p_0}{\sqrt{RT_{02}}}$$

$$F_{thrust} = \dot{m}u_e + A_e\left(p_e - p_a\right)$$

2. Chemical Rocket Propulsion

- Rocket thrust chamber – thrust



$$u_e = \sqrt{2\frac{\gamma}{\gamma-1}RT_{02}\left(1-\left(\frac{p_e}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right)}$$

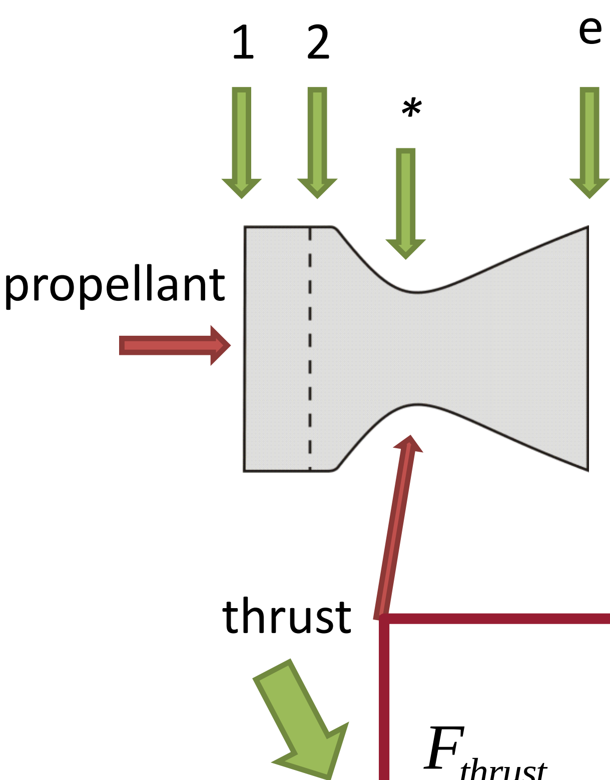
$$\dot{m} = \sqrt{\gamma\left(\frac{2}{1+\gamma}\right)^{\frac{1+\gamma}{(\gamma-1)}}}\frac{A^* p_0}{\sqrt{RT_{02}}}$$

$$\frac{F_{thrust}}{A^* p_0} = \sqrt{\left(\frac{2}{1+\gamma}\right)^{\frac{1+\gamma}{(\gamma-1)}}\frac{2\gamma^2}{\gamma-1}\left(1-\left(\frac{p_e}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right)} + \frac{A_e(p_e-p_a)}{A^* p_0}$$

2. Chemical Rocket Propulsion

Performance Characteristics

- Rocket thrust chamber – thrust



$$F_{thrust} = \dot{m}\frac{p_0 A^*}{\dot{m}}\frac{F_{thrust}}{p_0 A^*} =$$

$$= \dot{m} \; c^* \; C_F$$

characteristic velocity (c^*)

thrust coefficient (C_F)

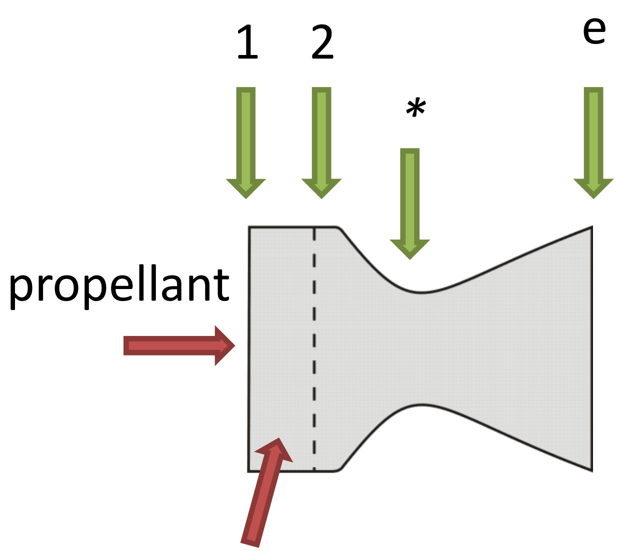
thrust depends only on stagnation pressure in combustion chamber

$$\frac{F_{thrust}}{A^* p_0} = \sqrt{\left(\frac{2}{1+\gamma}\right)^{\frac{1+\gamma}{(\gamma-1)}} \frac{2\gamma^2}{\gamma-1}\left[1-\left(\frac{p_e}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right]} + \frac{A_e\left(p_e - p_a\right)}{A^* p_0}$$

2. Chemical Rocket Propulsion

Performance Characteristics

- Rocket thrust chamber – characteristic velocity



characteristic velocity – c^*
specify combustion chamber

$$c^* = \frac{p_0 A^*}{\dot{m}}$$

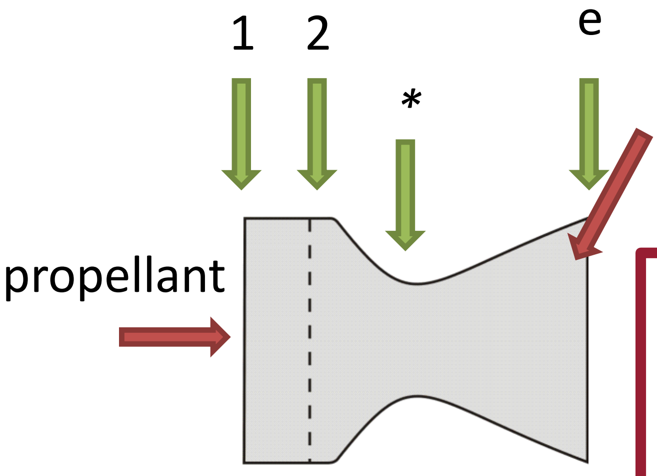
characteristic velocity is function of combustion chamber design and propellant characteristics

$$c^* = \sqrt{\frac{1}{\gamma}\left(\frac{\gamma+1}{2}\right)^{\frac{\gamma+1}{\gamma-1}} \frac{R_M T_{02}}{M_m}}$$

2. Chemical Rocket Propulsion

Performance Characteristics

- Rocket thrust chamber – thrust coefficient



thrust coefficient C_F → $C_F = \frac{F_{thrust}}{p_0 A^*}$

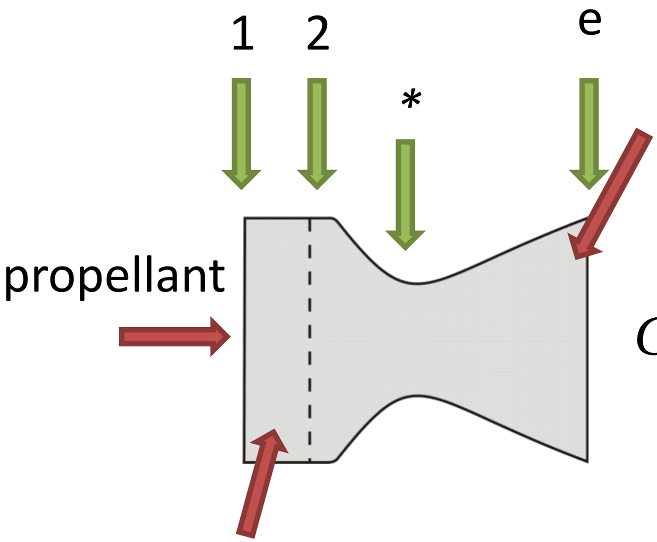
$$C_F = \sqrt{\left(\frac{2}{1+\gamma}\right)^{\frac{1+\gamma}{(\gamma-1)}} \frac{2\gamma^2}{\gamma-1}\left(1-\left(\frac{p_e}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right)} + \frac{A_e(p_e-p_a)}{A^* p_0}$$

thrust coefficient depends on gas property (γ) and nozzle parameters (nozzle area ratio and pressure ratio), it is independent on combustion chamber temperature

2. Chemical Rocket Propulsion

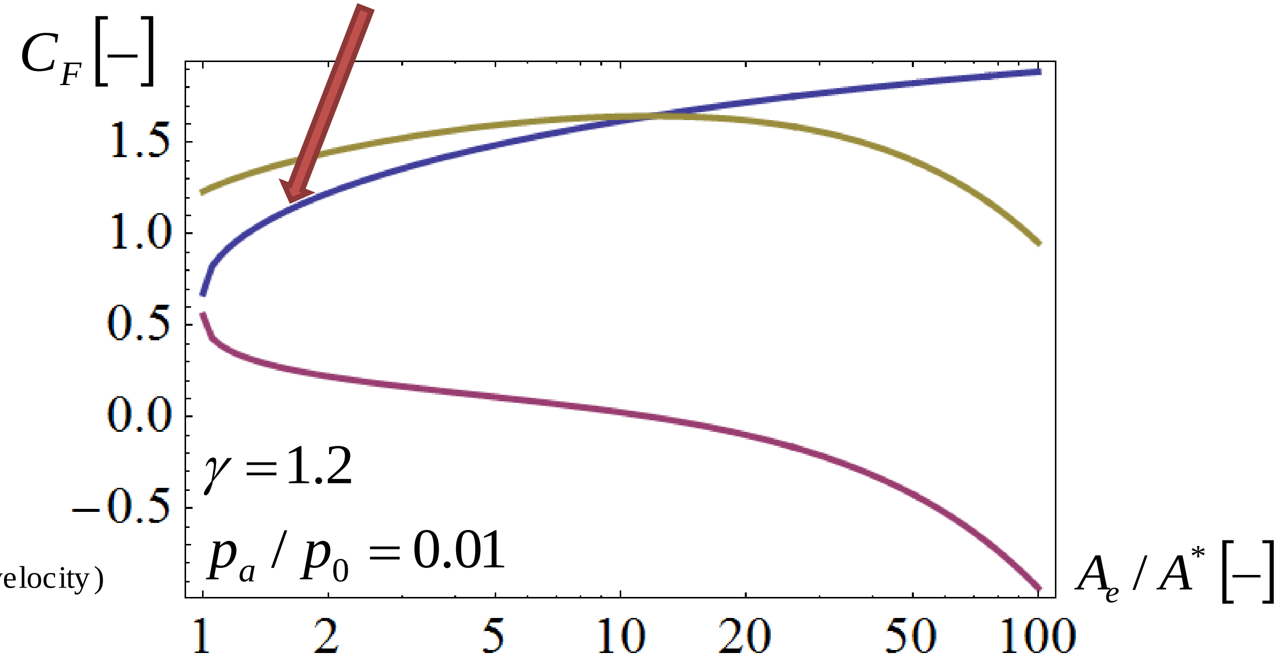
- Rocket thrust chamber – thrust coefficient

$$C_F = \sqrt{\left(\frac{2}{1+\gamma}\right)^{\frac{1+\gamma}{(\gamma-1)}} \frac{2\gamma^2}{\gamma-1}\left(1-\left(\frac{p_e}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right)} + \frac{A_e(p_e-p_a)}{A^* p_0}$$



contribution to thrust by exit velocity

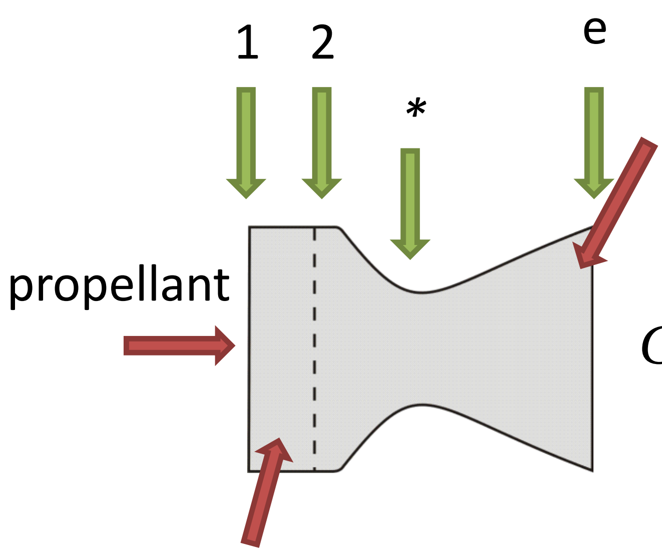
$$\frac{A_e}{A^*} \Rightarrow M_e \Rightarrow \frac{p_e}{p_0} \Rightarrow C_{F(\text{velocity})}$$



2. Chemical Rocket Propulsion

Performance Characteristics

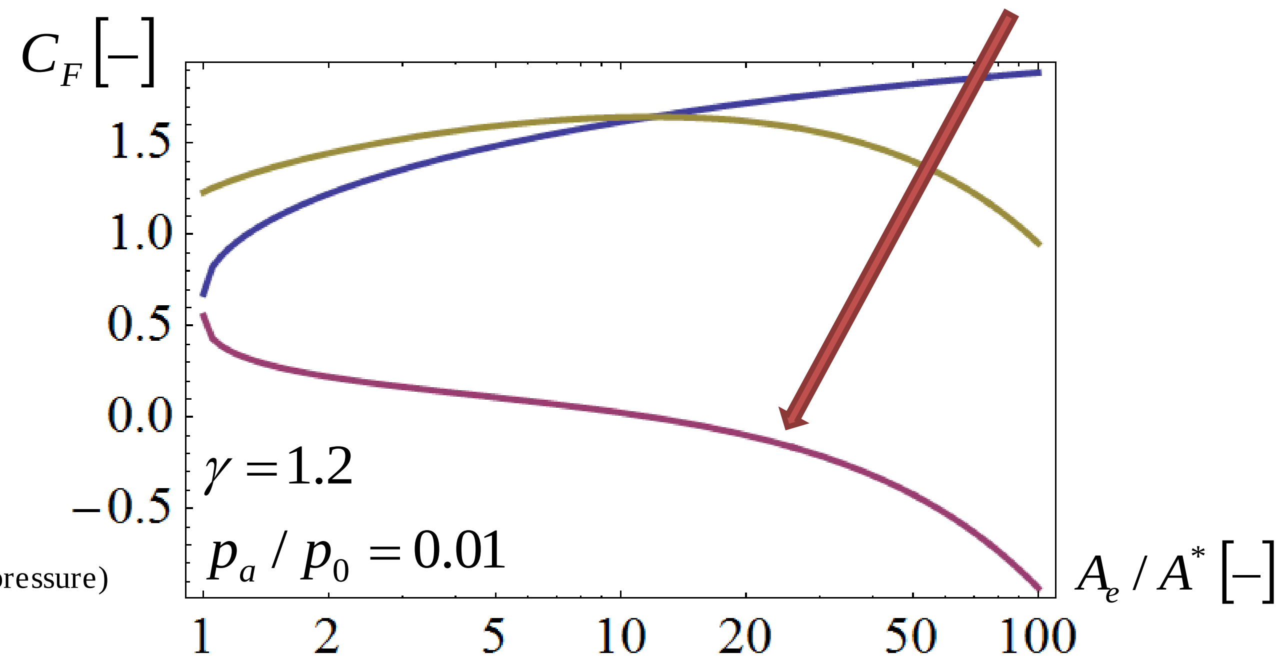
- Rocket thrust chamber – thrust coefficient



$$C_F = \sqrt{\left(\frac{2}{1+\gamma}\right)^{\frac{1+\gamma}{(\gamma-1)}} \frac{2\gamma^2}{\gamma-1}\left(1-\left(\frac{p_e}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right)} + \frac{A_e(p_e-p_a)}{A^* p_0}$$

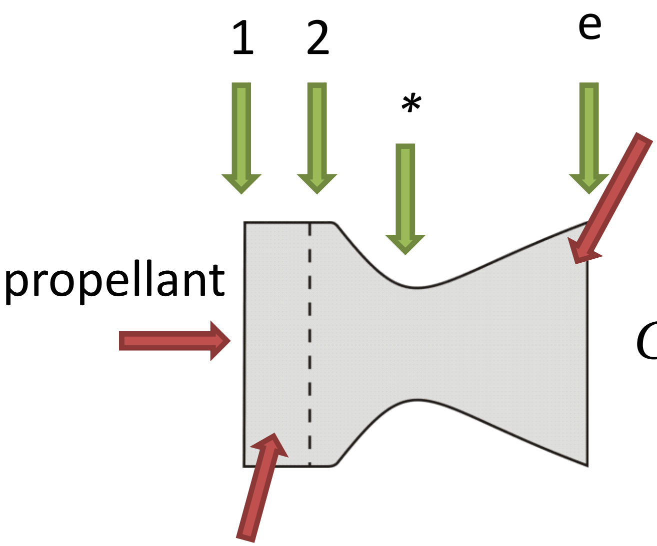
contribution to thrust by exit pressure

$$\frac{A_e}{A^*} \Rightarrow M_e \Rightarrow \frac{p_e}{p_0} \Rightarrow C_{F\,(\text{pressure})}$$



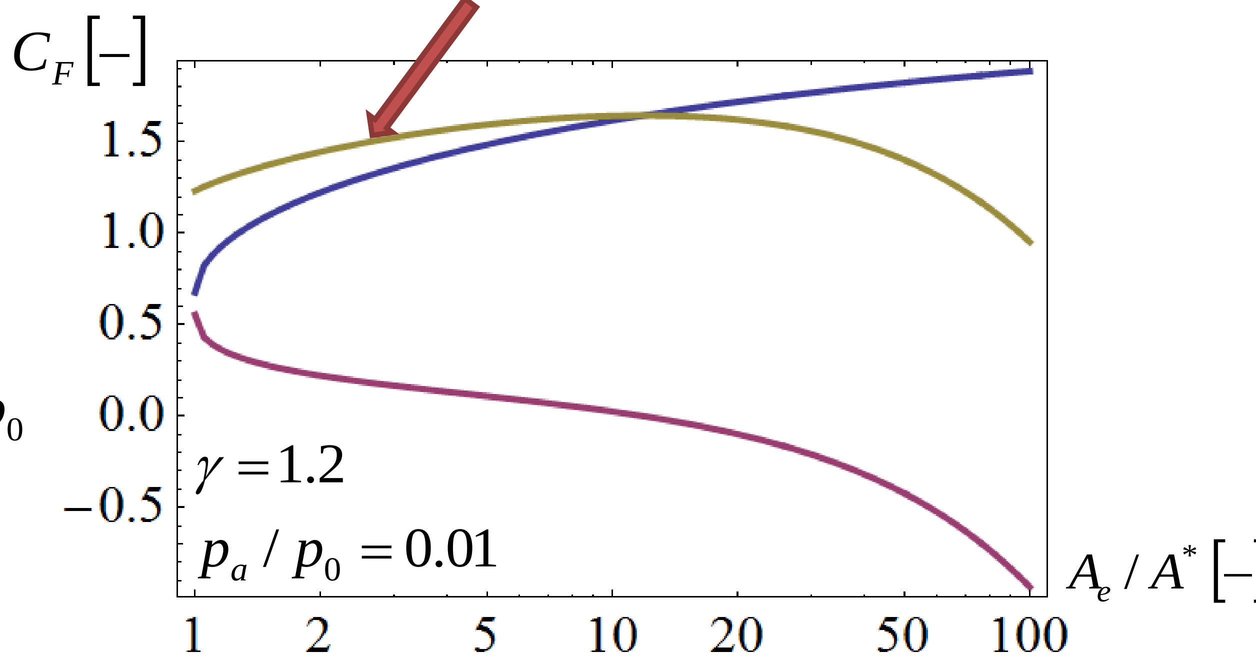
2. Chemical Rocket Propulsion

- Rocket thrust chamber – thrust coefficient



$$C_F = \sqrt{\left(\frac{2}{1+\gamma}\right)^{\frac{1+\gamma}{(\gamma-1)}} \frac{2\gamma^2}{\gamma-1}\left(1-\left(\frac{p_e}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right)} + \frac{A_e\left(p_e - p_a\right)}{A^* p_0}$$

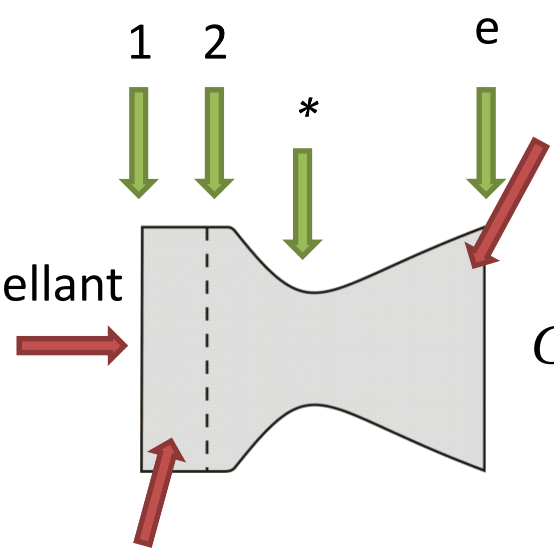
thrust coefficient for defined conditions p_a / p_0 depends on area ratio

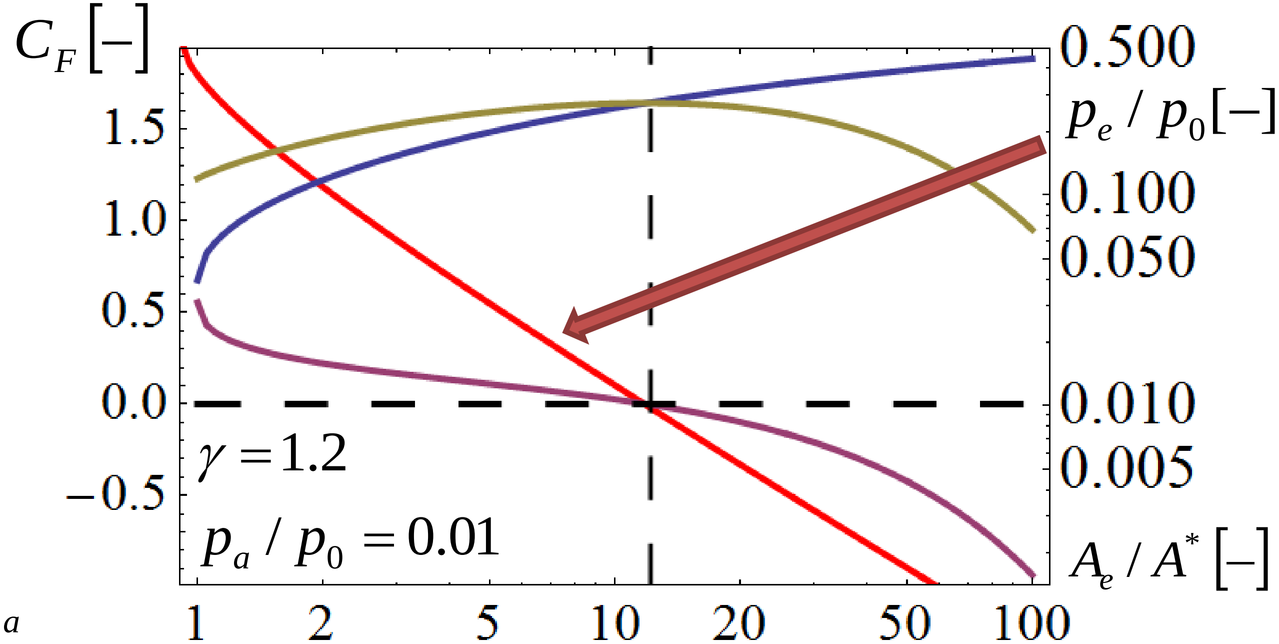


2. Chemical Rocket Propulsion

- Rocket thrust chamber – thrust coefficient

$$C_F = \sqrt{\left(\frac{2}{1+\gamma}\right)^{\frac{1+\gamma}{(\gamma-1)}} \frac{2\gamma^2}{\gamma-1}\left(1-\left(\frac{p_e}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right)} + \frac{A_e\left(p_e - p_a\right)}{A^* p_0}$$





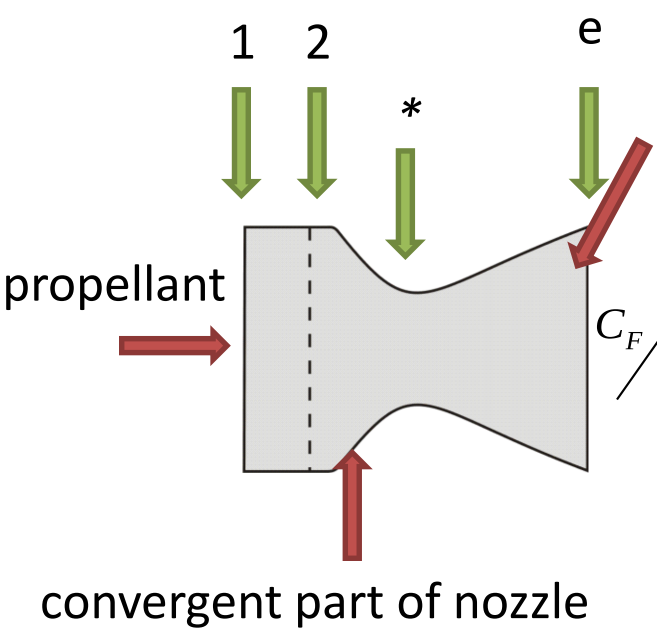
optimal thrust coefficient is defined by area ratio, where exit pressure equals ambient pressure $p_e = p_a$

2. Chemical Rocket Propulsion

Performance Characteristics

- Rocket thrust chamber – thrust coefficient

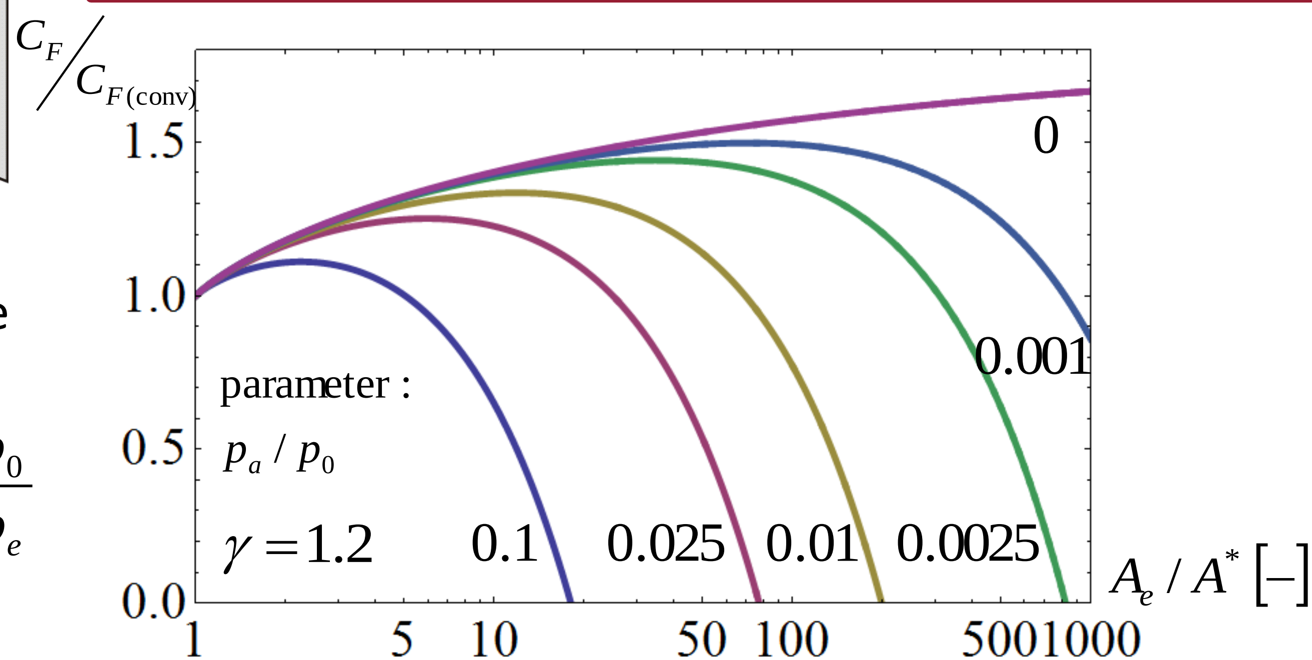
$$C_F = \sqrt{\left(\frac{2}{1+\gamma}\right)^{\frac{1+\gamma}{(\gamma-1)}} \frac{2\gamma^2}{\gamma-1}\left(1-\left(\frac{p_e}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right)} + \frac{A_e\left(p_e-p_a\right)}{A^* p_0}$$



$M=1$

$$\left(1+\frac{(\gamma-1)}{2}M^2\right)^{\frac{\gamma}{\gamma-1}} = \frac{p_0}{p_e}$$

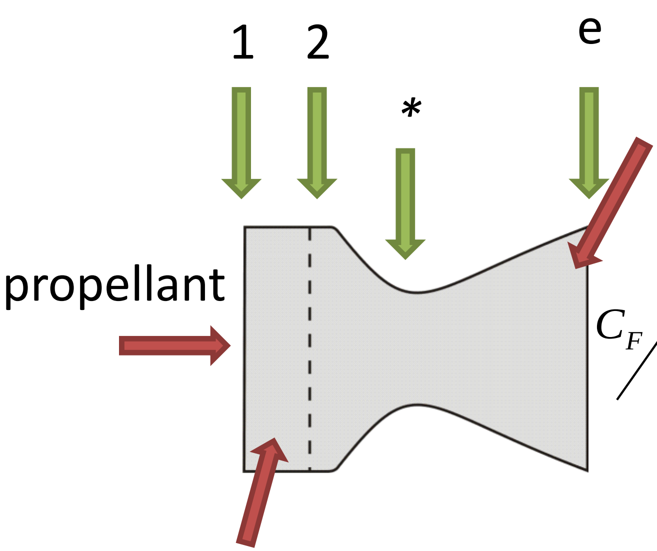
$C_{F\,(\mathrm{conv})} \leftarrow C_F$



2. Chemical Rocket Propulsion

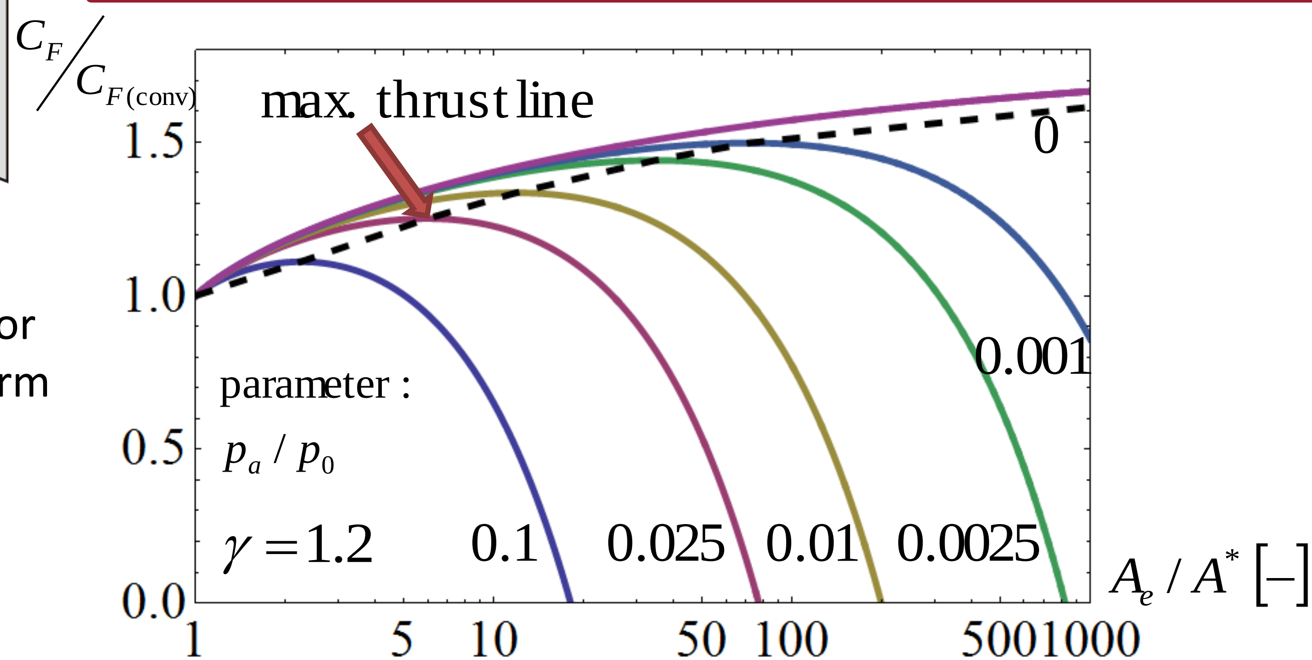
Performance Characteristics

- Rocket thrust chamber – thrust coefficient



$$C_F = \sqrt{\left(\frac{2}{1+\gamma}\right)^{\frac{1+\gamma}{(\gamma-1)}} \frac{2\gamma^2}{\gamma-1}\left(1-\left(\frac{p_e}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right)} + \frac{A_e\left(p_e - p_a\right)}{A^* p_0}$$

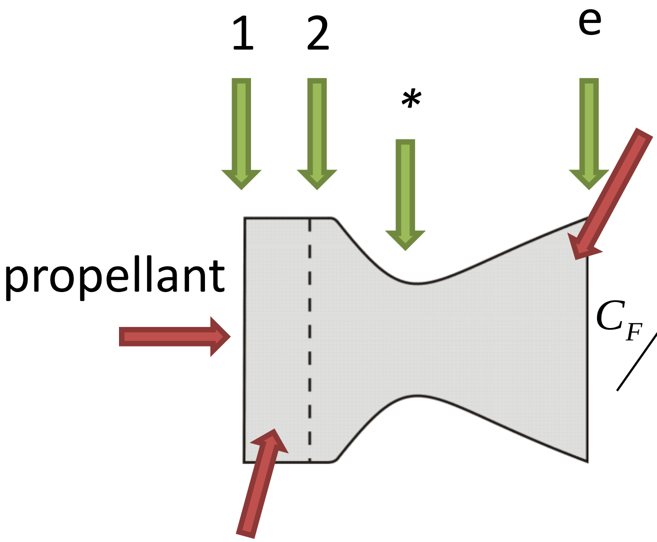
optimal thrust coefficient for individual pressure ratio form max. thrust line



2. Chemical Rocket Propulsion

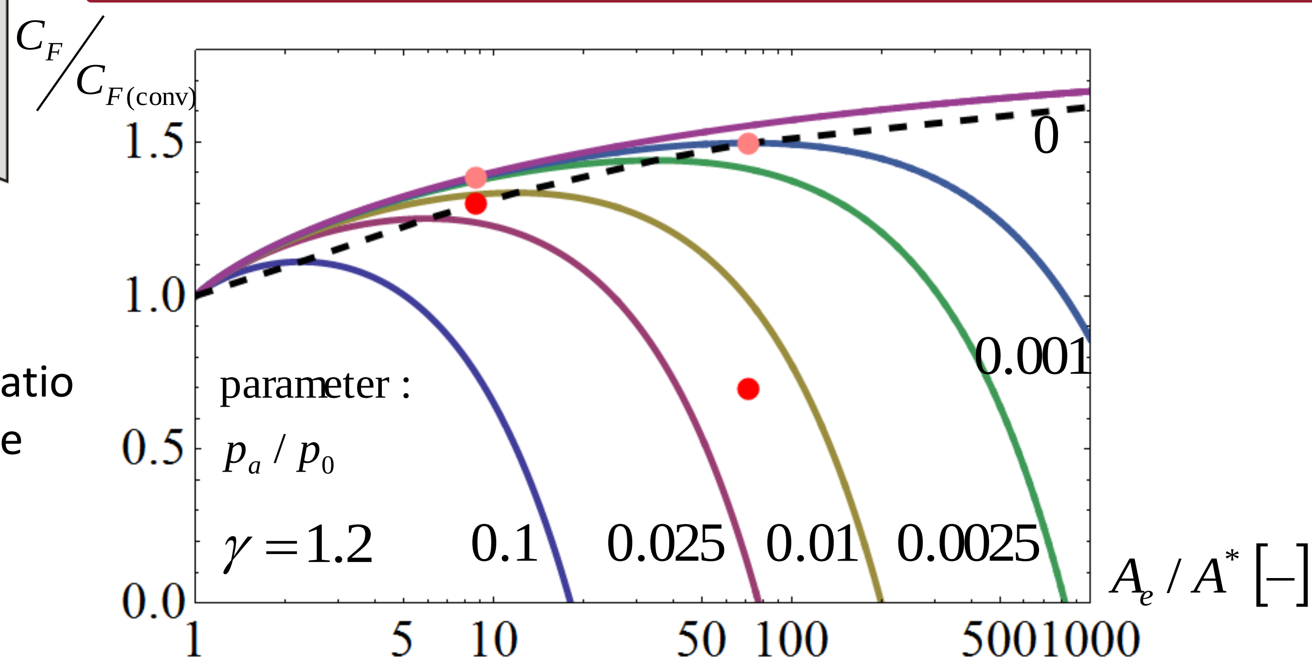
Performance Characteristics

- Rocket thrust chamber – thrust coefficient



$$C_F = \sqrt{\left(\frac{2}{1+\gamma}\right)^{\frac{1+\gamma}{(\gamma-1)}} \frac{2\gamma^2}{\gamma-1}\left(1-\left(\frac{p_e}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right)} + \frac{A_e(p_e-p_a)}{A^* p_0}$$

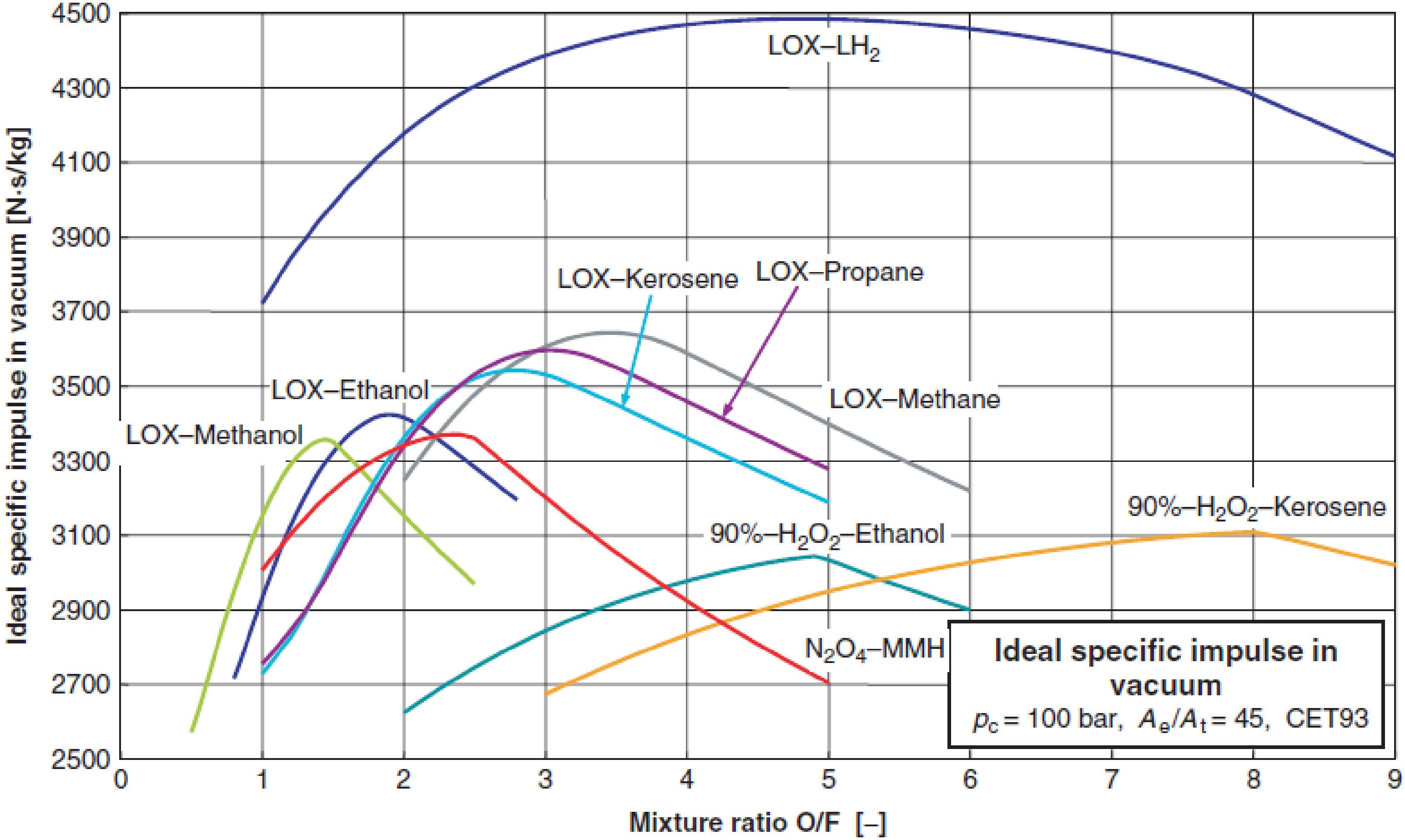
optimal thrust coefficient for sea level and pressure ratio 0.001 and the change of the coefficient



2. Chemical Rocket Propulsion

Liquid Propellant Performance

- performance of individual liquid propellants



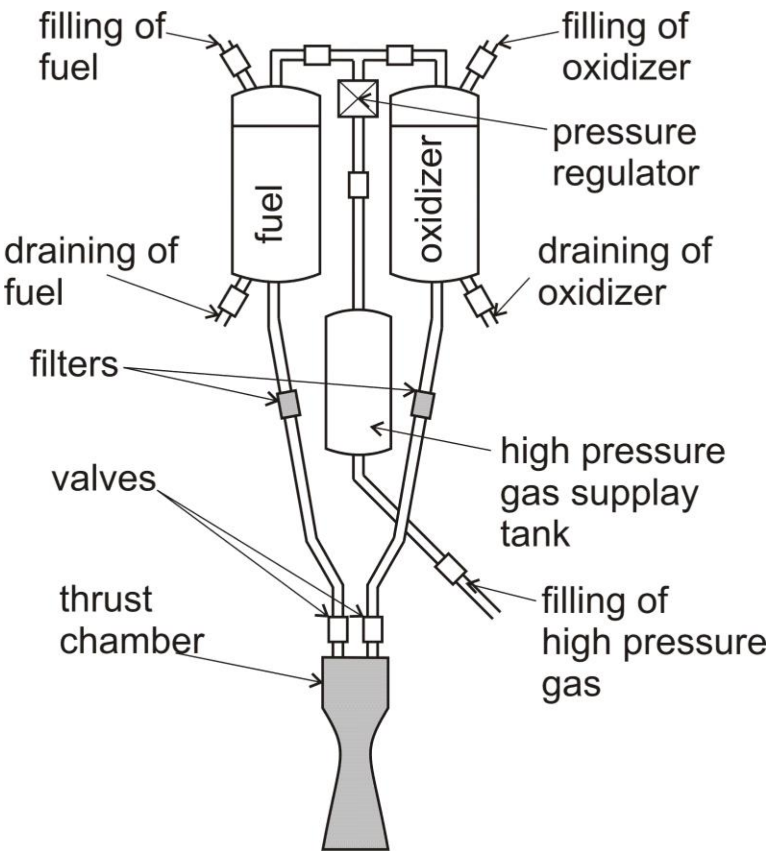
source: Ley, Wittmann, Hallmann: Handbook of space technology

2. Chemical Rocket Propulsion

Feed System

- There are 2 main feed systems for liquid propellant:

pressurized systems



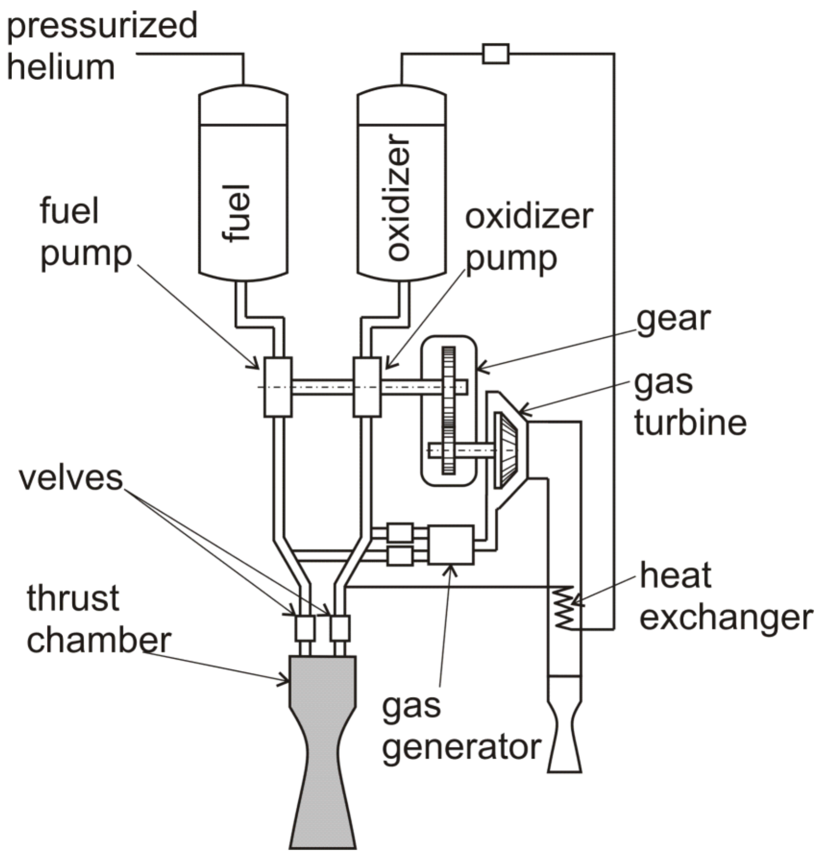
- They are usually used when:
  - total impulse is small
  - pressure in combustion chamber is small
- Disadvantages:
  - walls of tanks are thicker – system is heavier
- Usage:
  - control of attitude and change of orbit

2. Chemical Rocket Propulsion

Feed System

- There are 2 main feed systems for liquid propellant:

turbopump systems



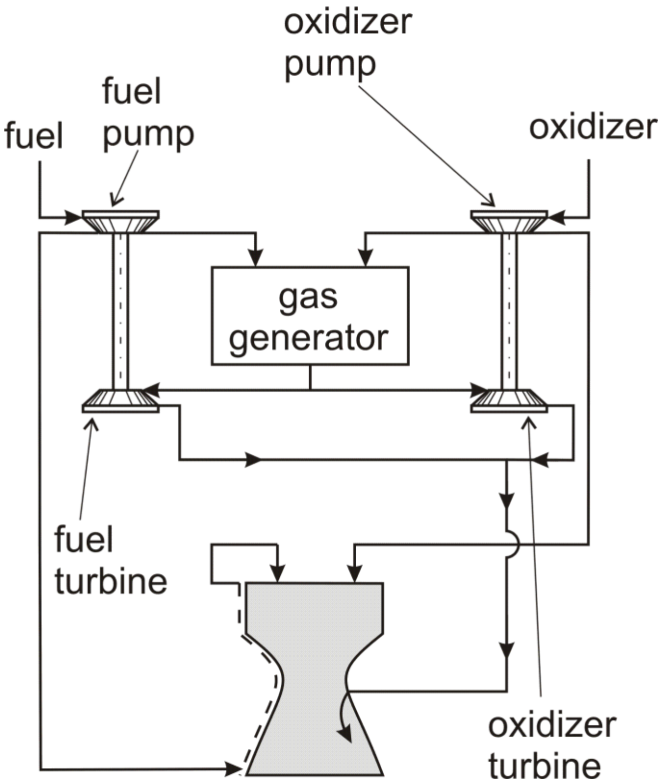
- They are usually used when:
  - total impulse is large
  - pressure in combustion chamber is large
- Positive characteristics of system:
  - pressure in tanks is lower than pressure in tanks when gas pressure feed system is used so the thickness of walls of tank is smaller
- Usage:
  - dominantly for boosters

2. Chemical Rocket Propulsion

Feed System

- turbopump systems – 3 basic cycles

Gas generator cycle - open cycle



Description:

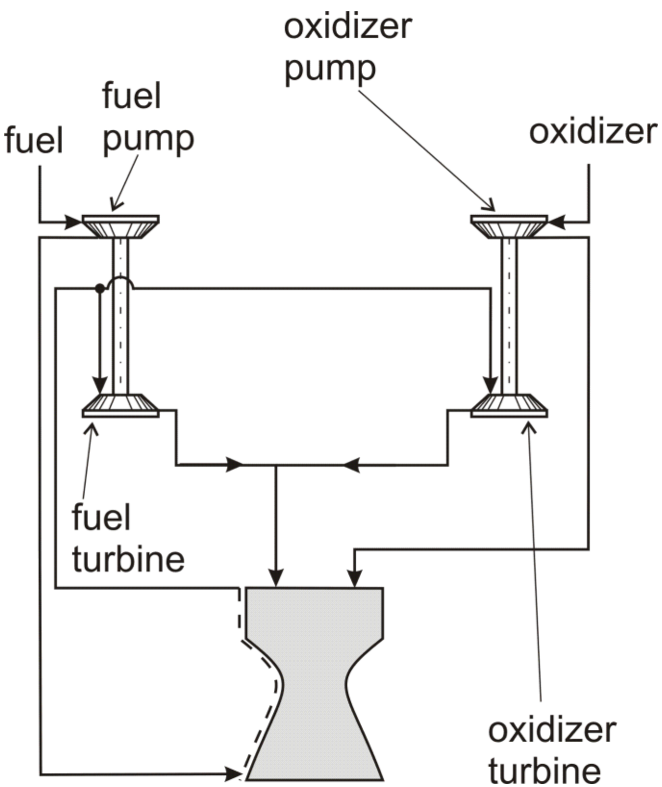
- It is the most common cycle
- It is relatively simple cycle
- The cycle efficiency is smaller than efficiency of closed cycle
- Small part of the propellant is consumed in small combustion chamber for generating gas for a turbine, which drives the pump
- Gas from turbine flows to separate nozzle or to the end part of the main nozzle, where it operates as cooler of nozzle
- Engines: F-1 (Saturn V) , 2 Vulcain (Ariane 5)

2. Chemical Rocket Propulsion

Feed System

- turbopump systems – 3 basic cycles

Expander cycle - closed cycle



Description:

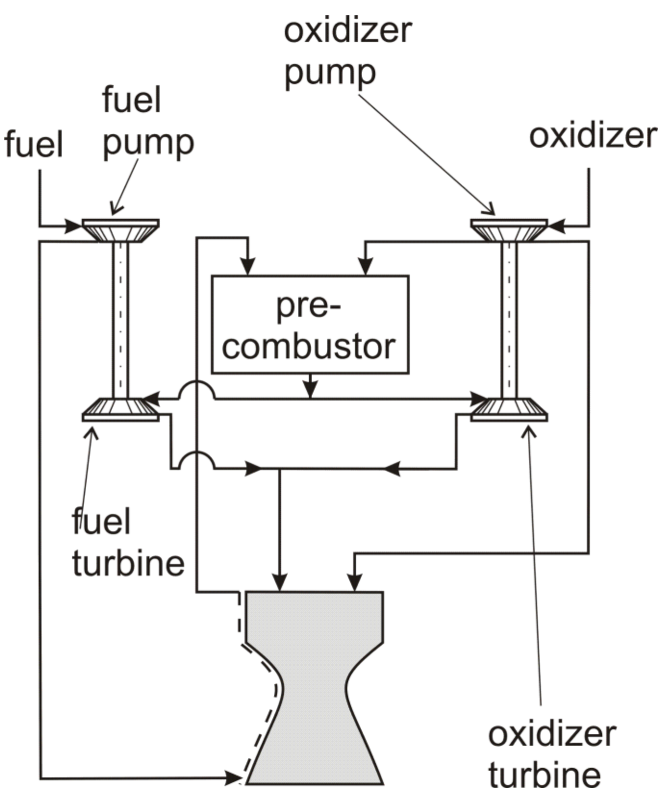
- The fuel passed through the cooling jacket of nozzle where it picked up energy and the fuel works as coolant of nozzle
- The fuel is evaporated, heated, and then fed to low pressure-ratio turbines
- at the outlet of the turbine fuel enters the combustion chamber where it is mixed with an oxidizer
- in that cycle all the fuel is burnt in combustion chamber and the efficiency of engine is increased
- Engines: RL10 (the second stage of the Delta IV) , Vinci (ESA)

2. Chemical Rocket Propulsion

Feed System

- turbopump systems – 3 basic cycles

Staged-combustion cycle - closed cycle



Description:

- The fuel passed through the cooling jacket of nozzle as in expander cycle
- Then the fuel flows into the precombustor where all the fuel is burnt with a part of the oxidizer, forming a high-energy gas to drive
- The turbines that drive the pumps all the gas at the outlet of the turbine flows into the combustion chamber where is mixed with remaining oxidizer
- pressure in combustion chamber: up to 40 MPa
- Engines: Space Shuttle Main Engine – SSME, RD-170 (Energija)

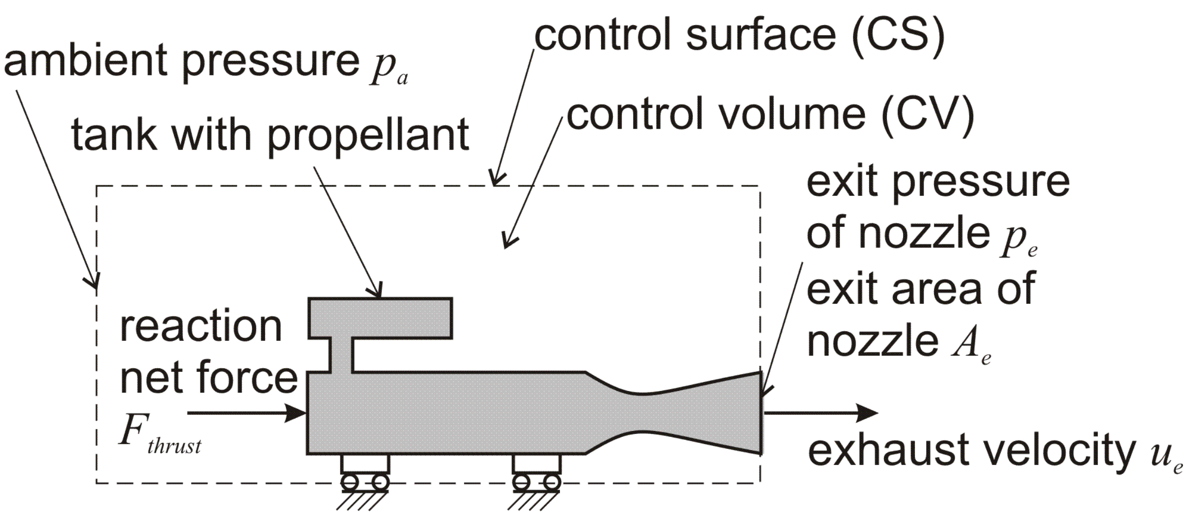
3. Performance of Rocket Vehicle

- Static Performance
- Force-Free Motion
- Motion with Gravity
- Launch Flight Mechanics

3. Performance of Rocket Vehicle

Static Performance

- Momentum equation – written for CV:
- simplifications:
  - quasi-one dimensional flow
  - steady-state flow

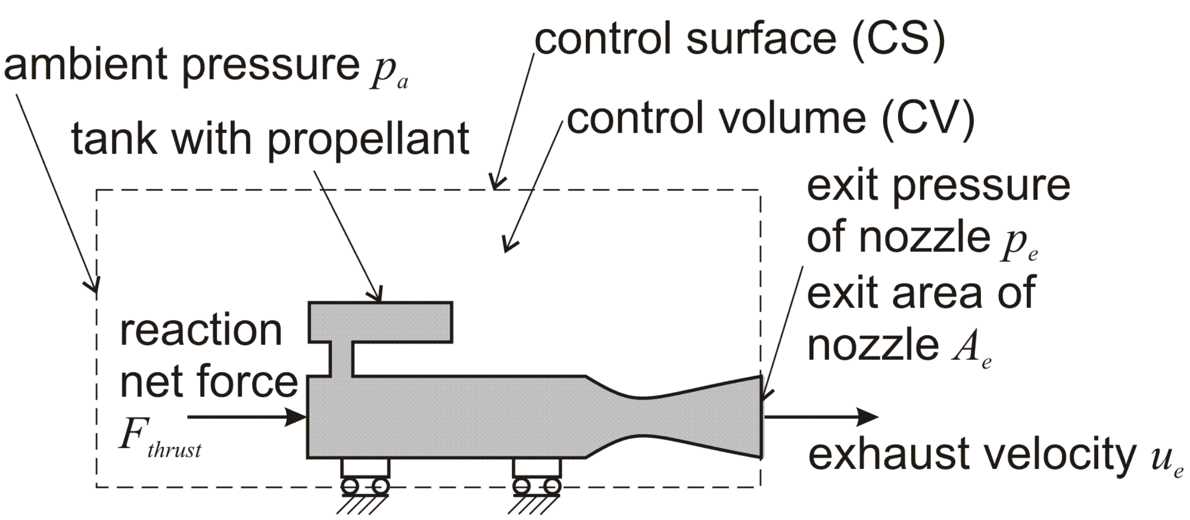


3. Performance of Rocket Vehicle

Static Performance

- Momentum equation – written for CV:

$$\begin{bmatrix} \text{Forces on} \\ \text{gas in CV in} \\ \text{direction } x \end{bmatrix} = \begin{bmatrix} \text{Rate} \\ \text{momentum} \\ \text{leaves CV} \end{bmatrix} - \begin{bmatrix} \text{Rate} \\ \text{momentum} \\ \text{enters CV} \end{bmatrix}$$

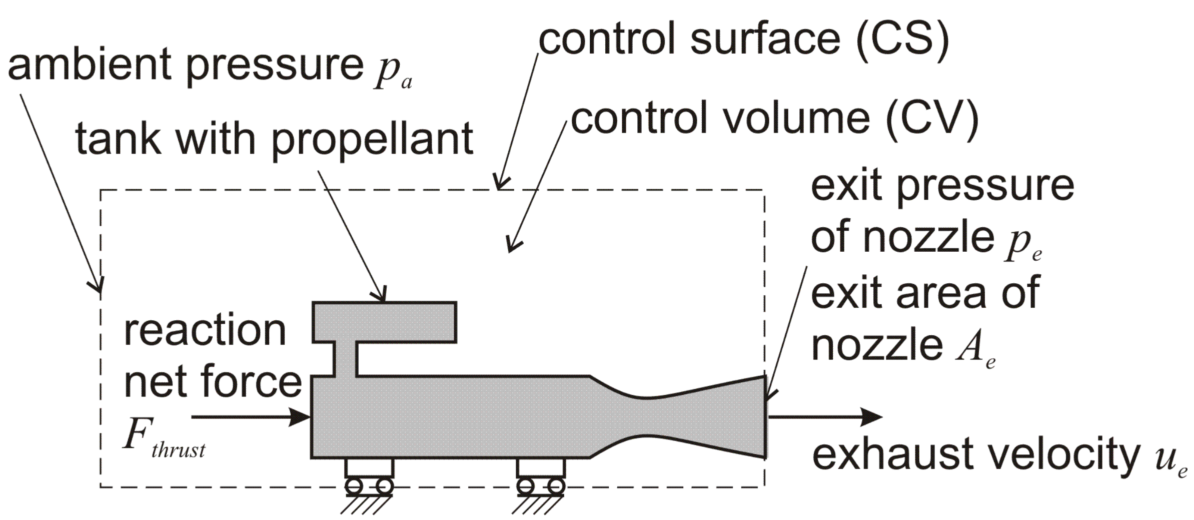


3. Performance of Rocket Vehicle

Static Performance

- Momentum equation – written for CV:

$$\begin{bmatrix}\text{Forces on}\\ \text{gas in CV in}\\ \text{direction } x\end{bmatrix} = \begin{bmatrix}\text{Rate}\\ \text{momentum}\\ \text{leaves CV}\end{bmatrix} - \begin{bmatrix}\text{Rate}\\ \text{momentum}\\ \text{enters CV}\end{bmatrix} \Rightarrow F_{thrust} + A_e\left(p_a - p_e\right) = \dot{m}u_e$$



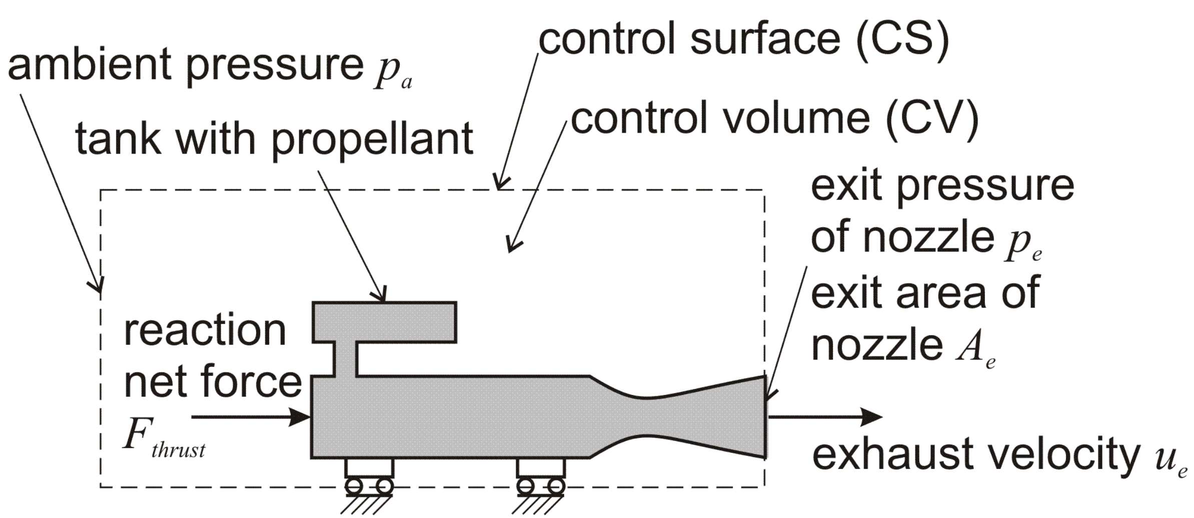
3. Performance of Rocket Vehicle

Static Performance

- Momentum equation – written for CV:

$$\begin{bmatrix}\text{Forces on}\\ \text{gas in CV in}\\ \text{direction } x\end{bmatrix} = \begin{bmatrix}\text{Rate}\\ \text{momentum}\\ \text{leaves CV}\end{bmatrix} - \begin{bmatrix}\text{Rate}\\ \text{momentum}\\ \text{enters CV}\end{bmatrix} \Rightarrow F_{thrust} + A_e\left(p_a - p_e\right) = \dot{m}u_e$$

$$F_{thrust} = \dot{m}u_e + A_e\left(p_e - p_a\right) = \dot{m}\left(u_e + A_e\left(\frac{p_e - p_a}{\dot{m}}\right)\right)$$



3. Performance of Rocket Vehicle

Static Performance

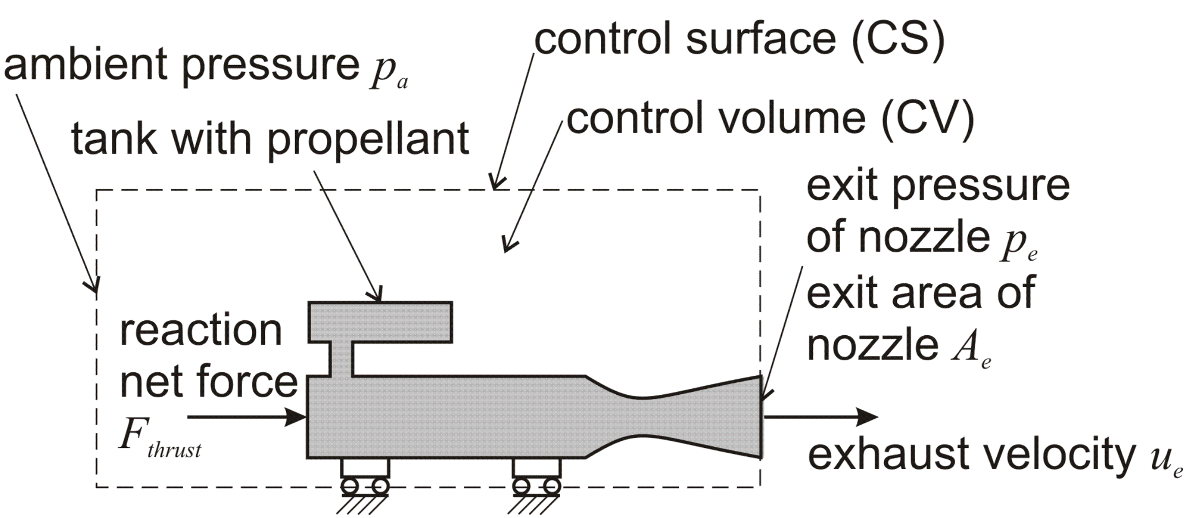
- Momentum equation – written for CV:

$$\begin{bmatrix}\text{Forces on}\\ \text{gas in CV in}\\ \text{direction } x\end{bmatrix} = \begin{bmatrix}\text{Rate}\\ \text{momentum}\\ \text{leaves CV}\end{bmatrix} - \begin{bmatrix}\text{Rate}\\ \text{momentum}\\ \text{enters CV}\end{bmatrix}$$

$$F_{thrust} + A_e(p_a - p_e) = \dot{m}u_e$$

$$F_{thrust} = \dot{m}u_e + A_e(p_e - p_a) =$$

$$= \dot{m}\left(u_e + A_e\left(\frac{p_e - p_a}{\dot{m}}\right)\right)$$



$$F_{thrust} = \dot{m}u_{ef}$$

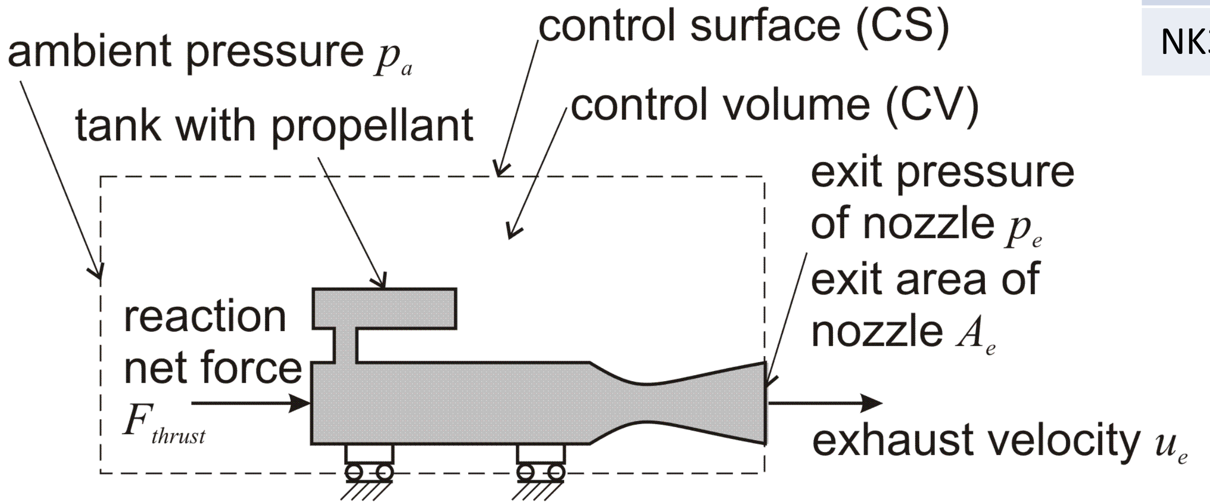
Thrust

3. Performance of Rocket Vehicle

Static Performance

- Momentum equation – written for CV:

Engine	Thrust [MN]
F1	7.77 (vacuum)
Vulcain 2	1.35
J2	1.03
NK33	1.51



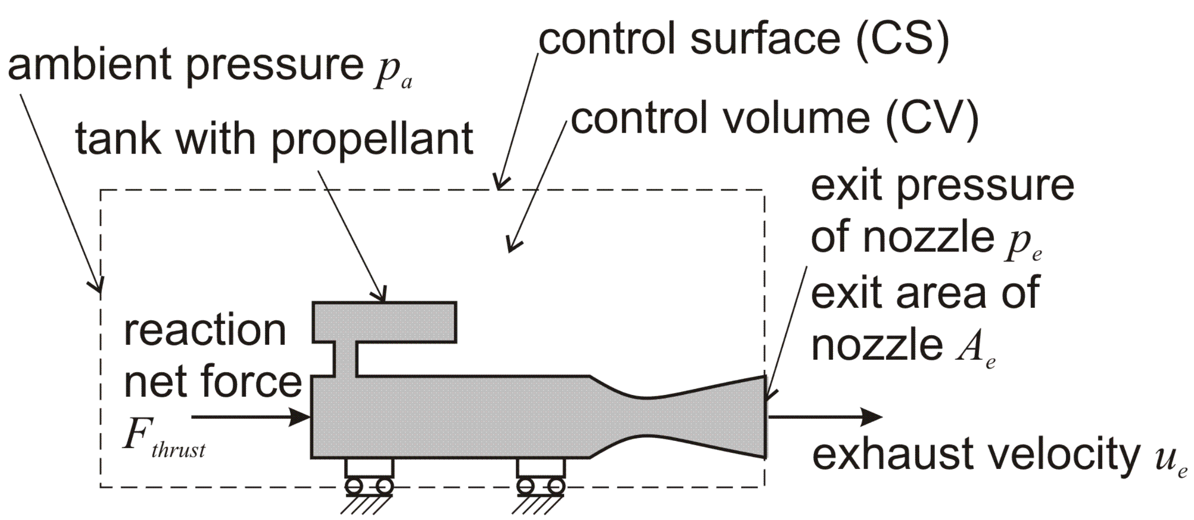
$$F_{thrust} = \dot{m} u_{ef}$$

Thrust

3. Performance of Rocket Vehicle

Static Performance

- Momentum equation – written for CV:



Total impulse

$$I_t = F_{thrust}\, t$$

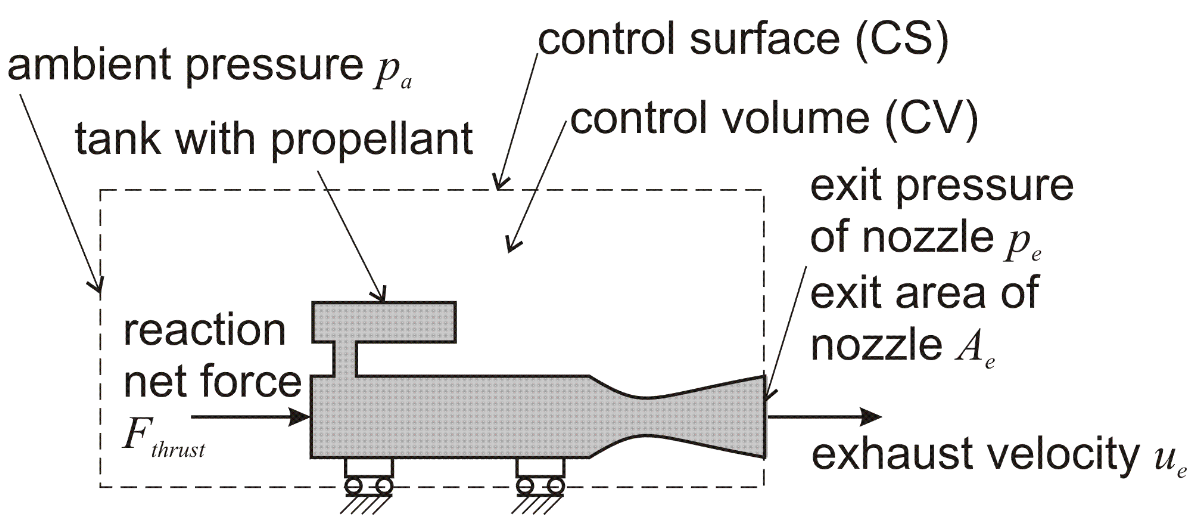
$$F_{thrust} = \dot{m} u_{ef}$$

Thrust

3. Performance of Rocket Vehicle

Static Performance

- Momentum equation – written for CV:



Specific impulse

$$I_s = I_t / mg = F_{thrust} / \dot{m}g =$$
$$= \dot{m}u_{ef} / \dot{m}g = u_{ef} / g$$

Total impulse

$$I_t = F_{thrust}\, t$$

$$F_{thrust} = \dot{m}u_{ef}$$

Thrust

3. Performance of Rocket Vehicle

Static Performance

- Momentum equation – written for CV:

Propellant	Specific impulse I_s [s]
cold gas	50
Monopropellant hydrazine	230
LOX/LH2	455
Ion propulsion	>3000

Specific impulse

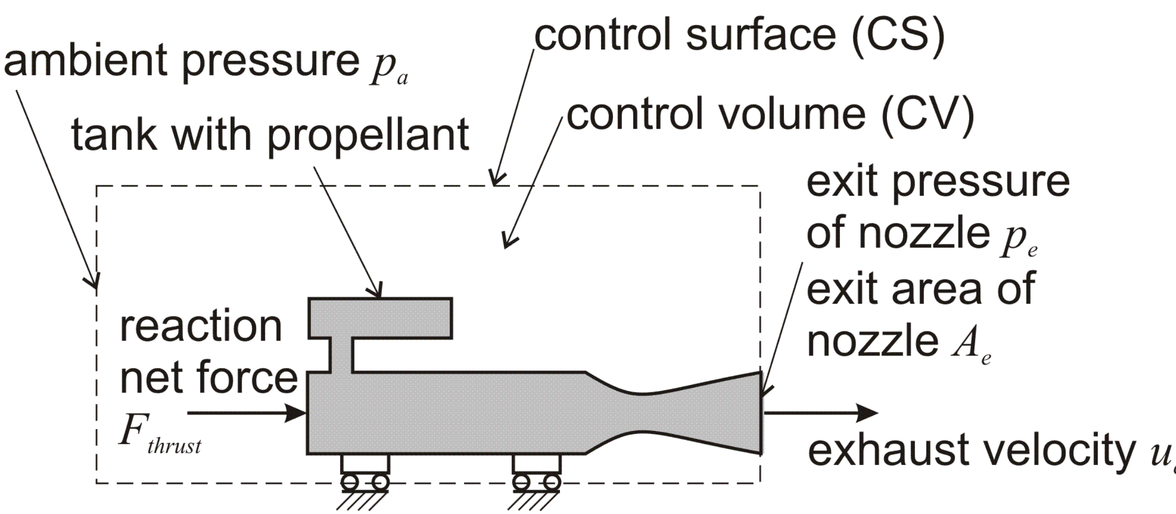
$$I_s = I_t / mg = F_{thrust} / \dot{m}g = \dot{m}u_{ef} / \dot{m}g = u_{ef} / g$$

Total impulse

$$I_t = F_{thrust}\, t$$

$$F_{thrust} = \dot{m}u_{ef}$$

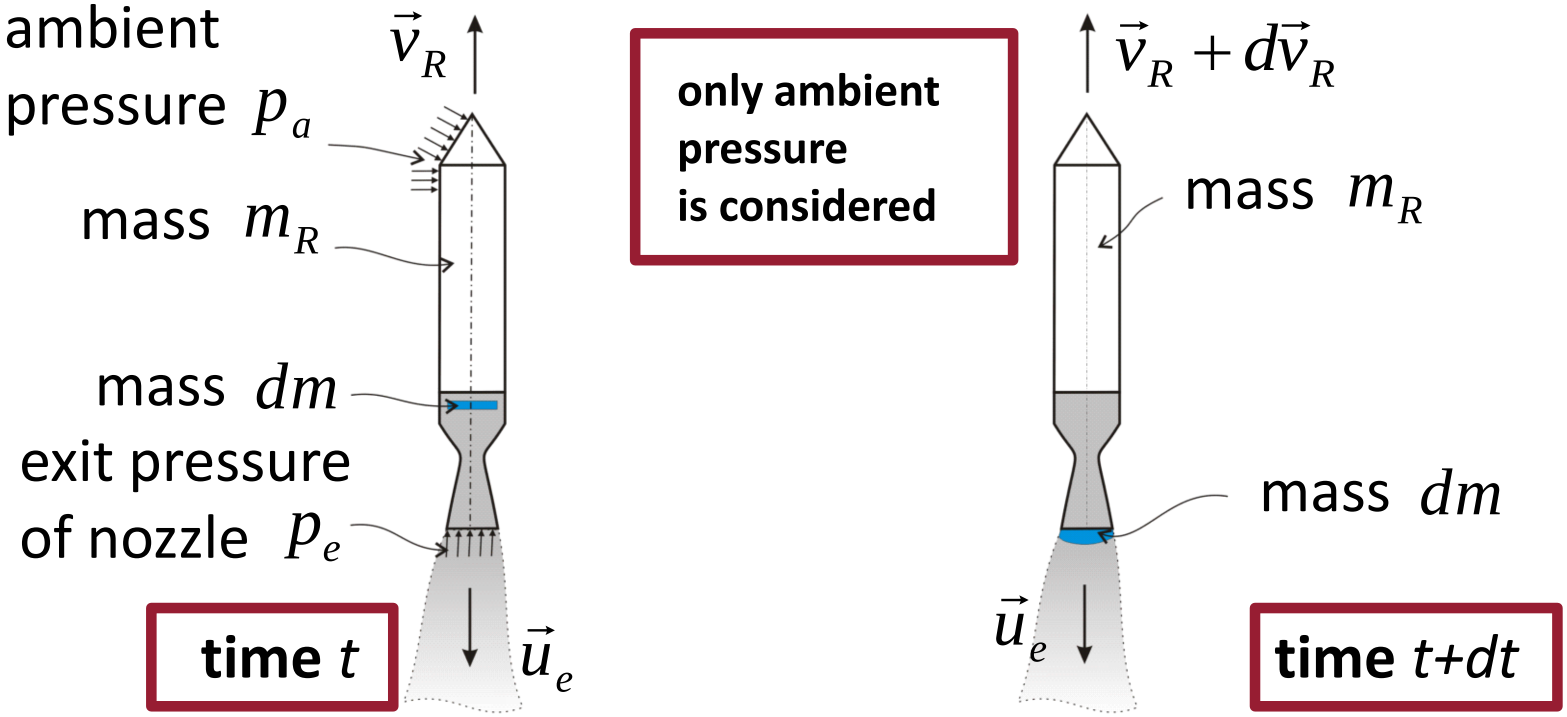
Thrust



3. Performance of Rocket Vehicle

Force-Free Motion

- Force-free motion ➡ absence of external forces



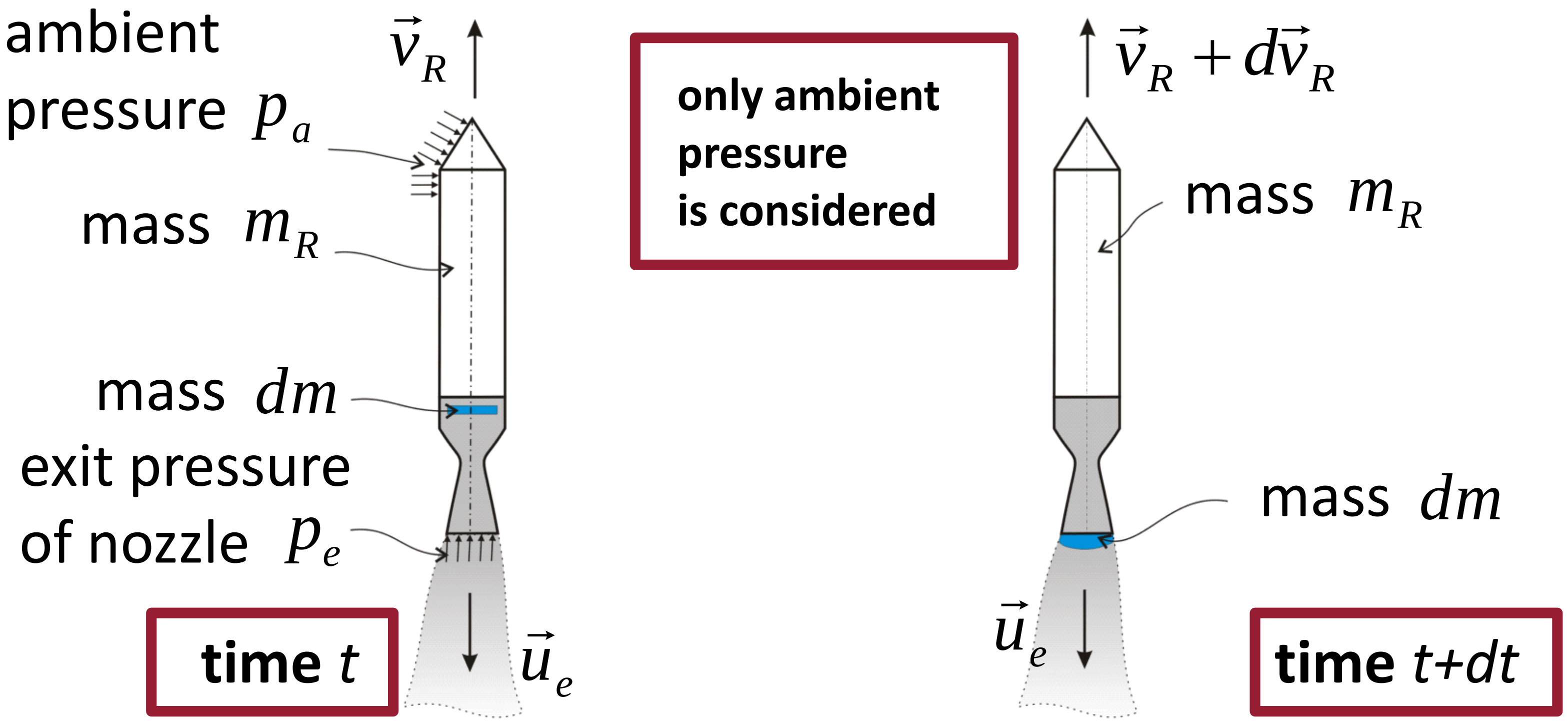
Momentum: $(m_R + dm)\vec{v}_R$ ➡ $m_R\left(\vec{v}_R + d\vec{v}_R\right) + dm\left(\vec{v}_R + \vec{u}_e\right)$

Change of momentum in dt $\quad m_R\, d\vec{v}_R + dm\vec{u}_e$

3. Performance of Rocket Vehicle

Force-Free Motion

- Force-free motion ⇒ absence of external forces



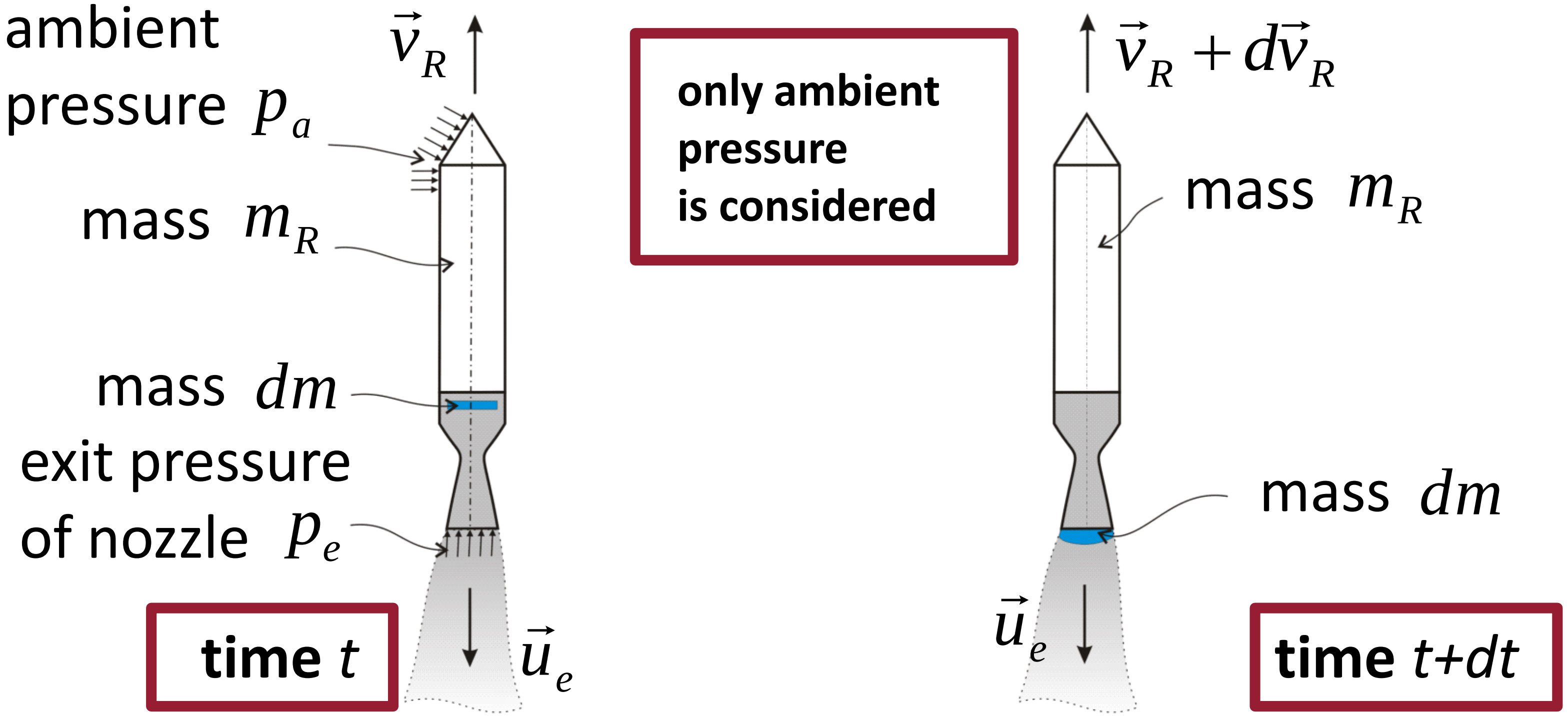
Pressure force: $(p_e - p_a)A_e\vec{i}_R$ ⇒ $(p_e - p_a)A_e\vec{i}_R$

Total impulse in dt : $(p_e - p_a)A_e\vec{i}_R dt$

3. Performance of Rocket Vehicle

Force-Free Motion

- Force-free motion ➡ absence of external forces



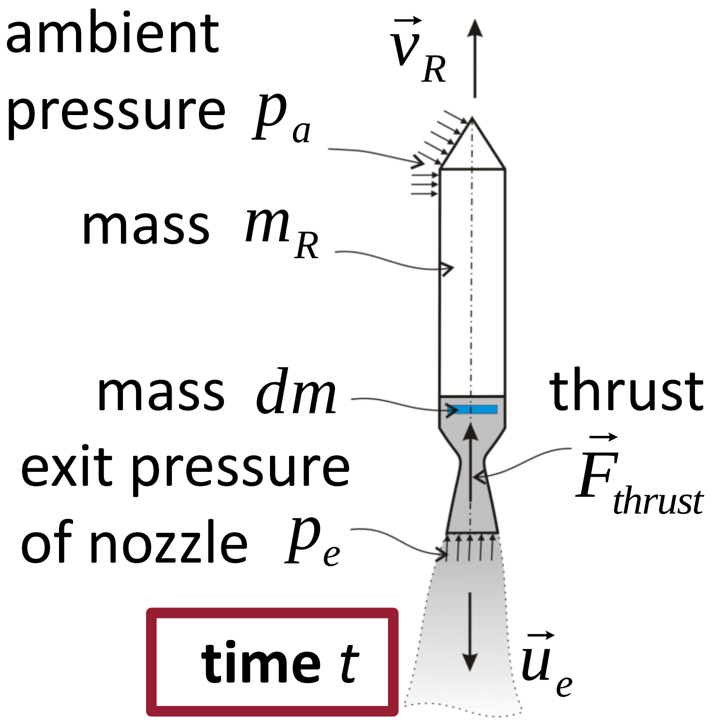
Momentum equation: $m_R\, d\vec{v}_R + dm\vec{u}_e = (p_e - p_a)A_e\vec{i}_R dt$

Momentum equation in $\vec{i}_R$: $m_R\, dv_R = dm u_e + (p_e - p_a)A_e dt$

3. Performance of Rocket Vehicle

Force-Free Motion

- Force-free motion ➡ absence of external forces



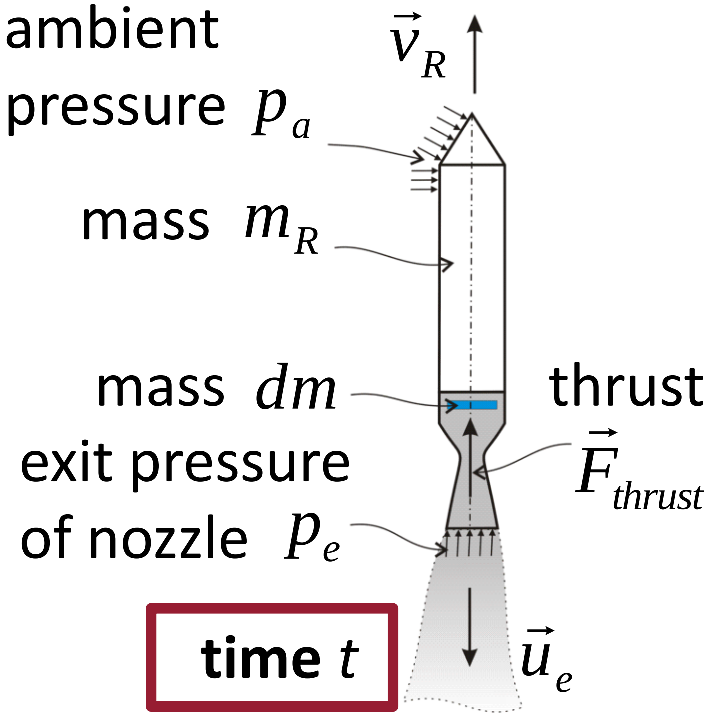
$$dm = \dot{m}dt$$

Momentum equation in $\vec{i}_R$: $m_R\, dv_R = dm u_e + (p_e - p_a)A_e dt$

3. Performance of Rocket Vehicle

Force-Free Motion

- Force-free motion ➡ absence of external forces



$$dm = \dot{m} dt$$

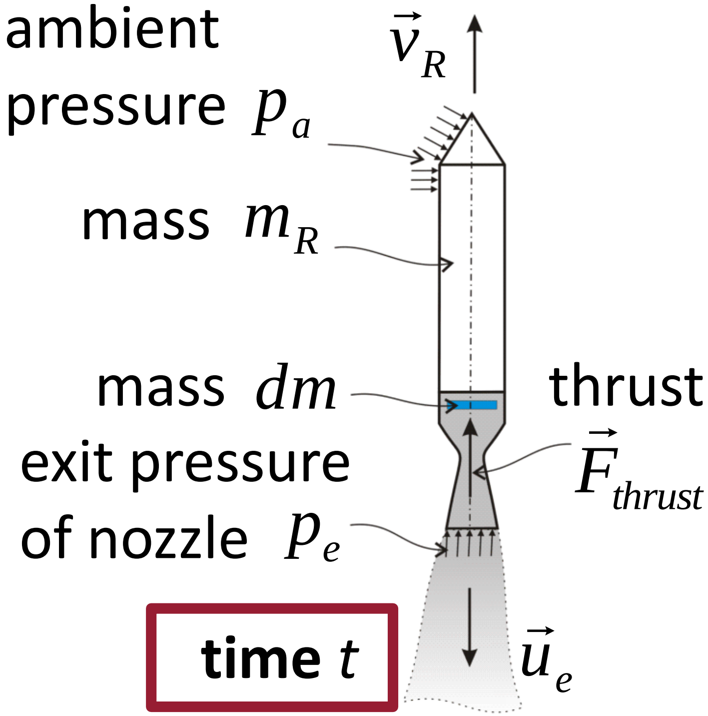
$$m_R \, dv_R = \left(\dot{m} u_e + \left(p_e - p_a \right) A_e \right) dt$$

Momentum equation in $\vec{i}_R$: $m_R \, dv_R = dm u_e + \left(p_e - p_a \right) A_e dt$

3. Performance of Rocket Vehicle

Force-Free Motion

- Force-free motion ➡ absence of external forces



time t

$$dm = \dot{m} dt$$

Momentum equation in $\vec{i}_R$: $m_R\, dv_R = dm u_e + (p_e - p_a) A_e dt$

$$m_R\, dv_R = (\dot{m} u_e + (p_e - p_a) A_e) dt$$

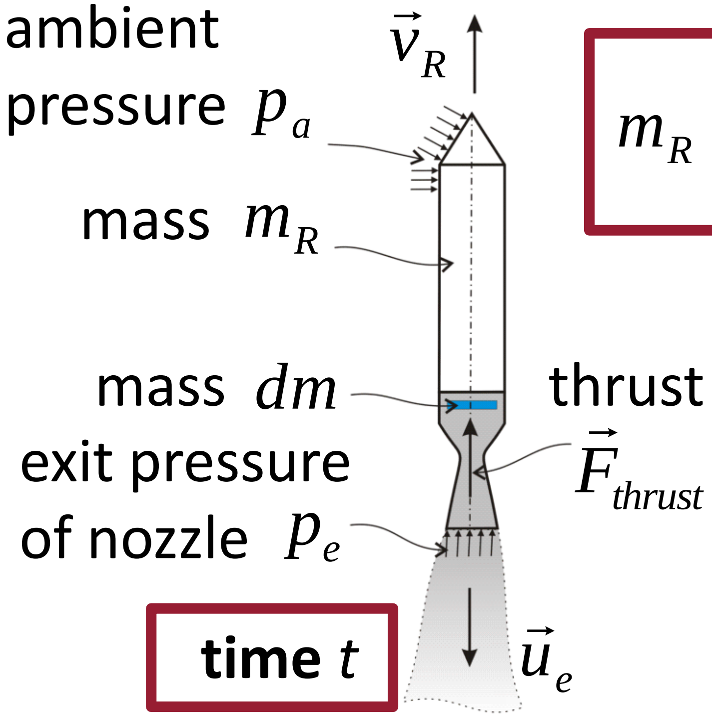
$$m_R\, dv_R = \dot{m} u_{ef} dt$$

$$m_R \frac{dv_R}{dt} = F_{thrust}$$

3. Performance of Rocket Vehicle

Force-Free Motion

- Force-free motion ⇒ absence of external forces



time t

$$m_R \frac{d\vec{v}_R}{dt} = \vec{F}_{thrust}$$

$$m_R \frac{dv_R}{dt} = F_{thrust}$$

$$m_R\, dv_R = \dot{m} u_{ef}\, dt$$

$$m_R\, dv_R = \left(\dot{m} u_e + \left(p_e - p_a\right) A_e\right) dt$$

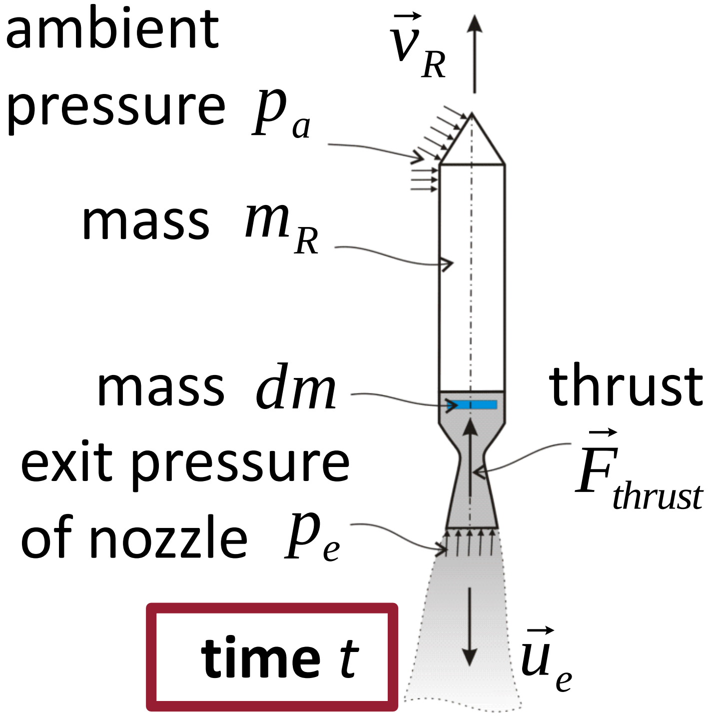
$$dm = \dot{m} dt$$

Momentum equation in $\vec{i}_R$: $m_R\, dv_R = dm u_e + \left(p_e - p_a\right) A_e dt$

3. Performance of Rocket Vehicle

Force-Free Motion

- Force-free motion ⇒ absence of external forces



time t

$$dv_R = -\frac{dm_R}{m_R} u_{ef}$$

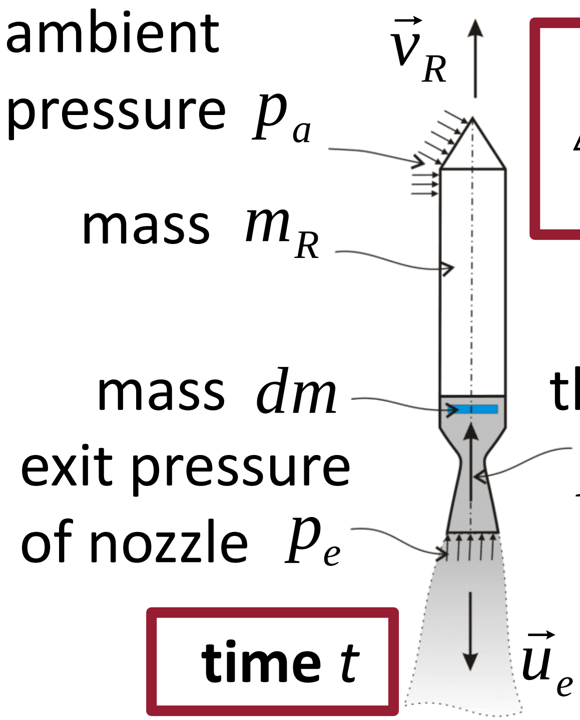
$$m_R \, dv_R = \dot{m} u_{ef} dt$$

$$\frac{dm_R}{dt} = -\dot{m}$$

3. Performance of Rocket Vehicle

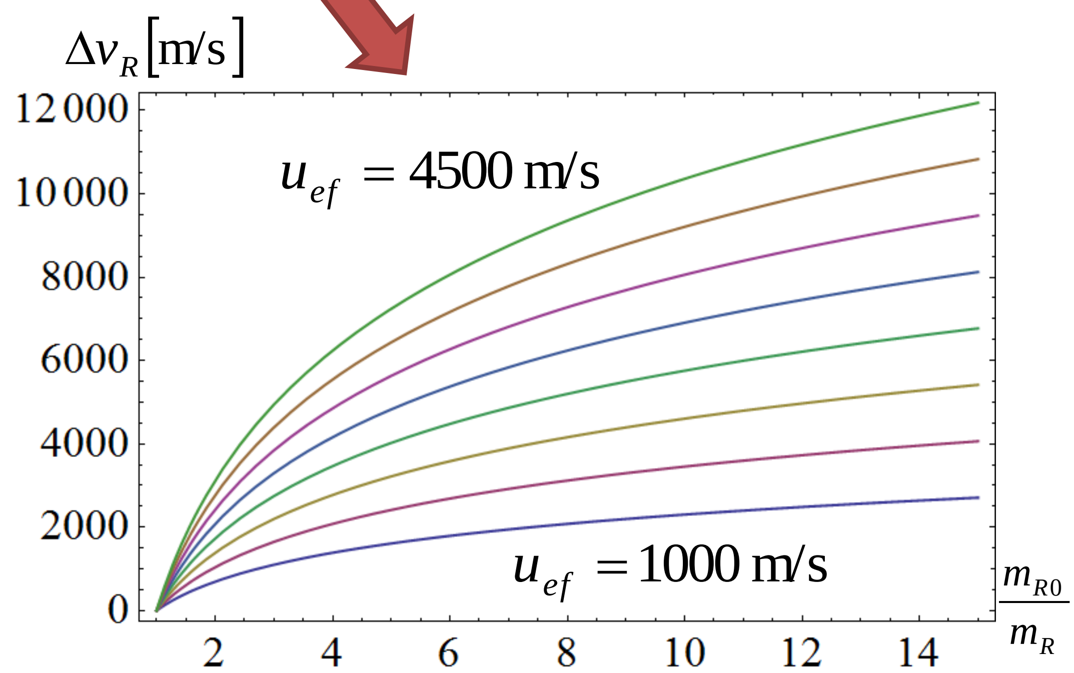
Force-Free Motion

- Force-free motion ➡ absence of external forces



time t

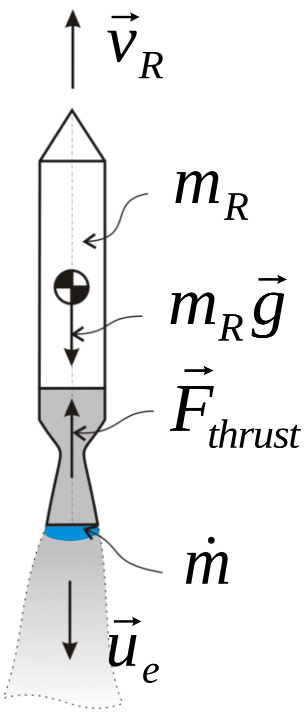
$$\Delta v_R = u_{ef} \,\mathrm{Ln}\left(\frac{m_{R0}}{m_R}\right) \Leftarrow dv_R = -\frac{dm_R}{m_R} u_{ef}$$



3. Performance of Rocket Vehicle

Motion with Gravity

- Motion with gravity ➡ vertical motion in gravity field



$$m_R \frac{dv_R}{dt} = -\frac{dm_R}{dt} u_{ef} - m_R g$$

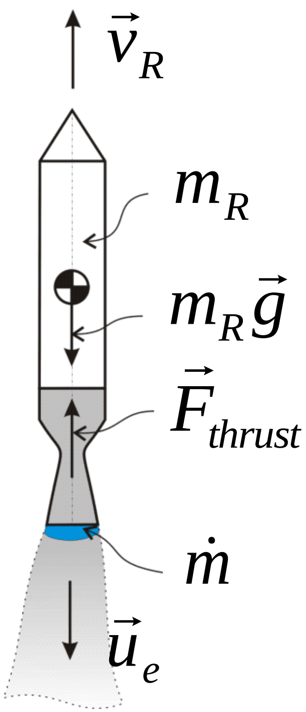
⬆

$$m_R \frac{d\vec{v}_R}{dt} = \frac{dm_R}{dt} \vec{u}_{ef} + m_R \vec{g}$$

3. Performance of Rocket Vehicle

Motion with Gravity

- Motion with gravity ⇨ vertical motion in gravity field



$$dv_R = -\frac{dm_R}{m_R}u_{ef} - g\,dt$$

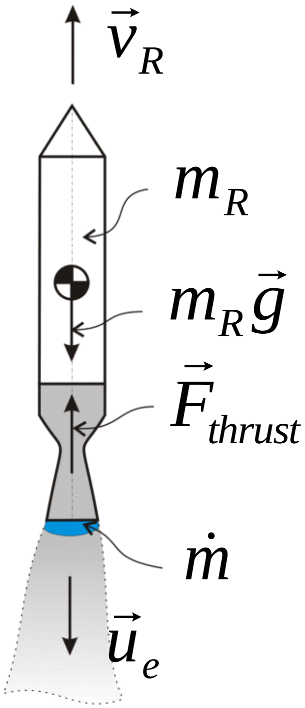
$$m_R\frac{dv_R}{dt} = -\frac{dm_R}{dt}u_{ef} - m_R g$$

$$m_R\frac{d\vec{v}_R}{dt} = \frac{dm_R}{dt}\vec{u}_{ef} + m_R\vec{g}$$

3. Performance of Rocket Vehicle

Motion with Gravity

- Motion with gravity ➡ vertical motion in gravity field



$$m_R \frac{d\vec{v}_R}{dt} = \frac{dm_R}{dt}\vec{u}_{ef} + m_R\vec{g}$$

⬆

$$m_R \frac{dv_R}{dt} = -\frac{dm_R}{dt}u_{ef} - m_R g$$

⬆

$$dv_R = -\frac{dm_R}{m_R}u_{ef} - g\,dt$$

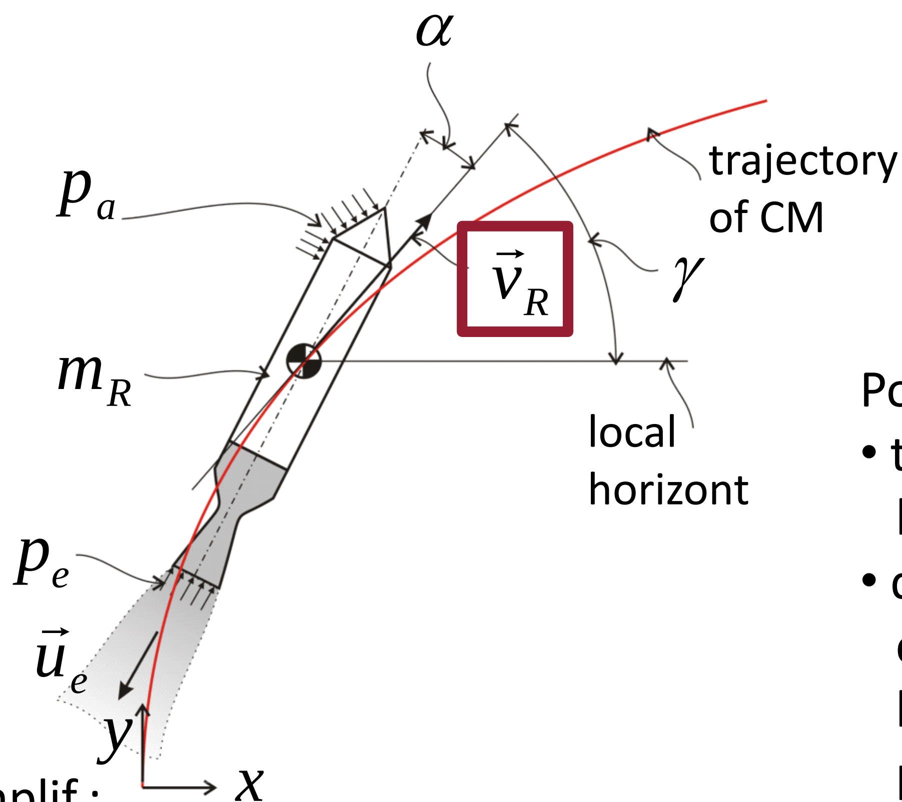
➡

$$\Delta v_R = u_{ef}\,\mathrm{Ln}\left(\frac{m_{R0}}{m_R}\right) - gt$$

3. Performance of Rocket Vehicle

Launch Flight Mechanics

- Forces act on rocket



Simplif.:
- all forces act on the same plane
- Earth is inertial frame of reference

The flight of rocket has 2 main phases:
1. powered phase
2. unpowered phase

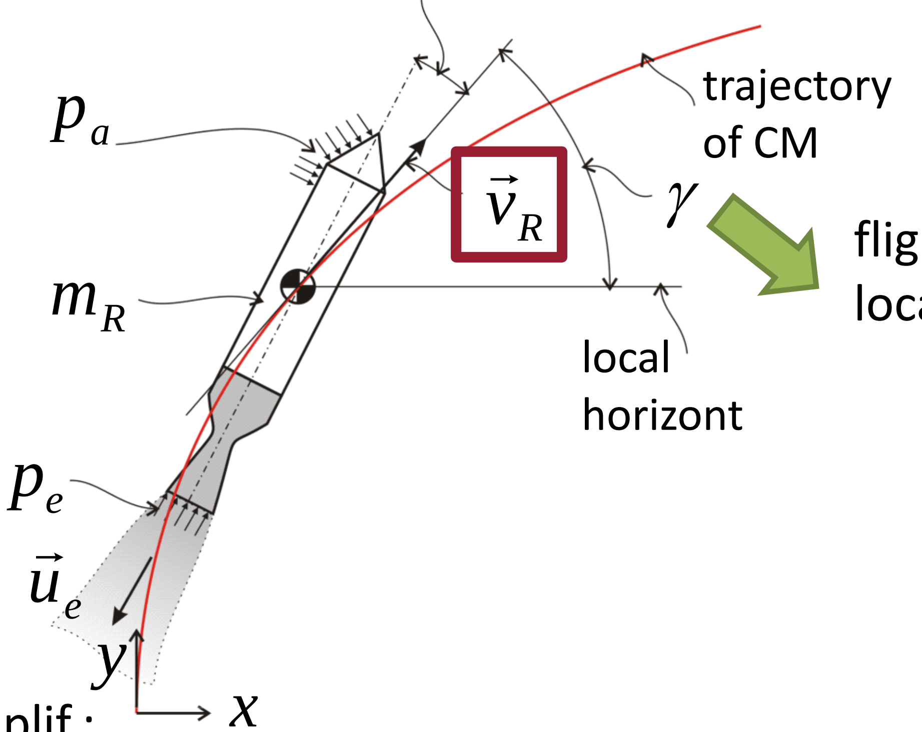
Powered phase:
- trajectory of vehicle from launch pad to burnout point
- during the phase, guidance system control the trajectory – vehicle at burnout point should have prescribed position and velocity

3. Performance of Rocket Vehicle

Launch Flight Mechanics

- Forces act on rocket

α → angle of attack



γ → flight path angle – angle between local horizont and velocity vector

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3. Performance of Rocket Vehicle

Launch Flight Mechanics

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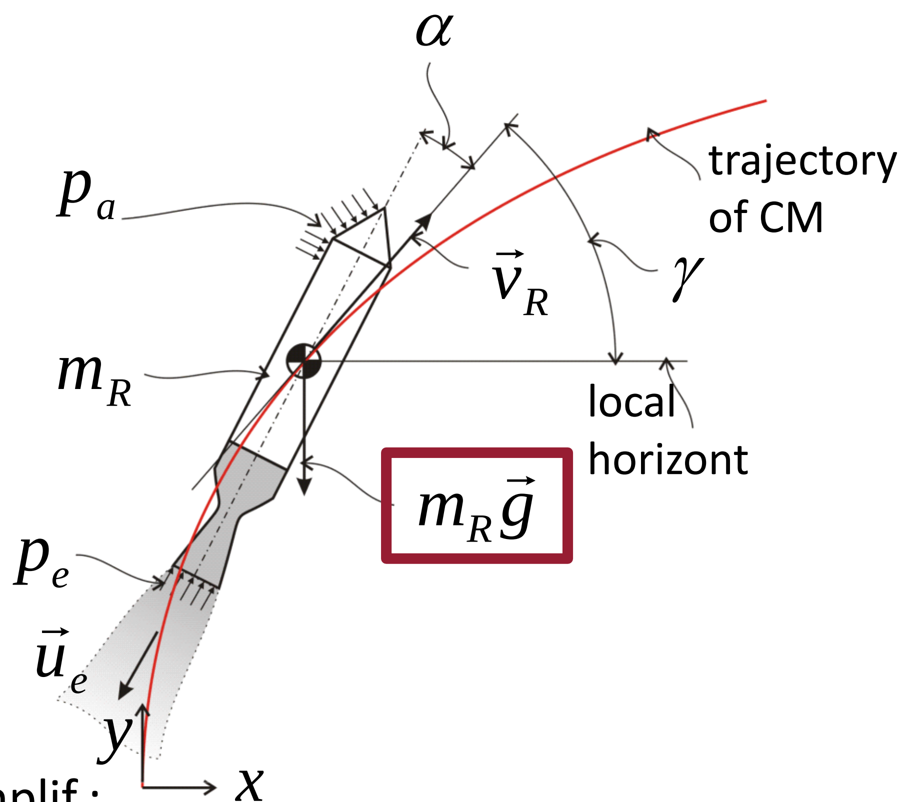

Simplif.:
- all forces act on the same plane
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Three forces act on rocket at each instant:

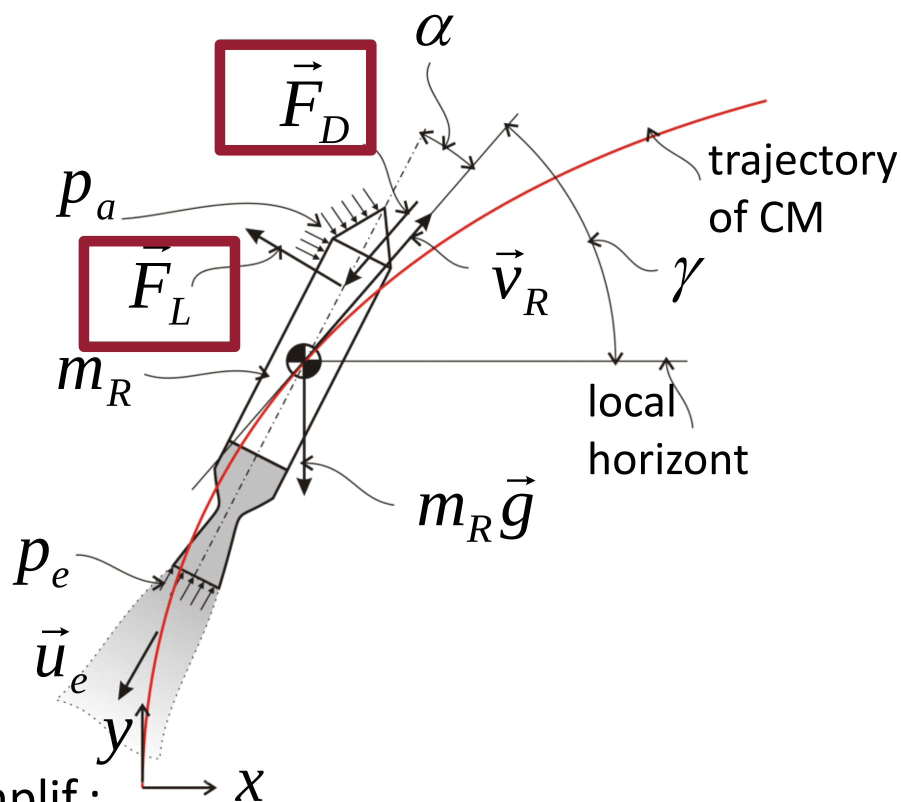
1. gravitational force – applied at the CM $m_R\vec{g}$

it is function of vertical location of rocket
mass of rocket is function of propellant mass flow

equation of propellant mass flow

3. Performance of Rocket Vehicle

- Forces act on rocket



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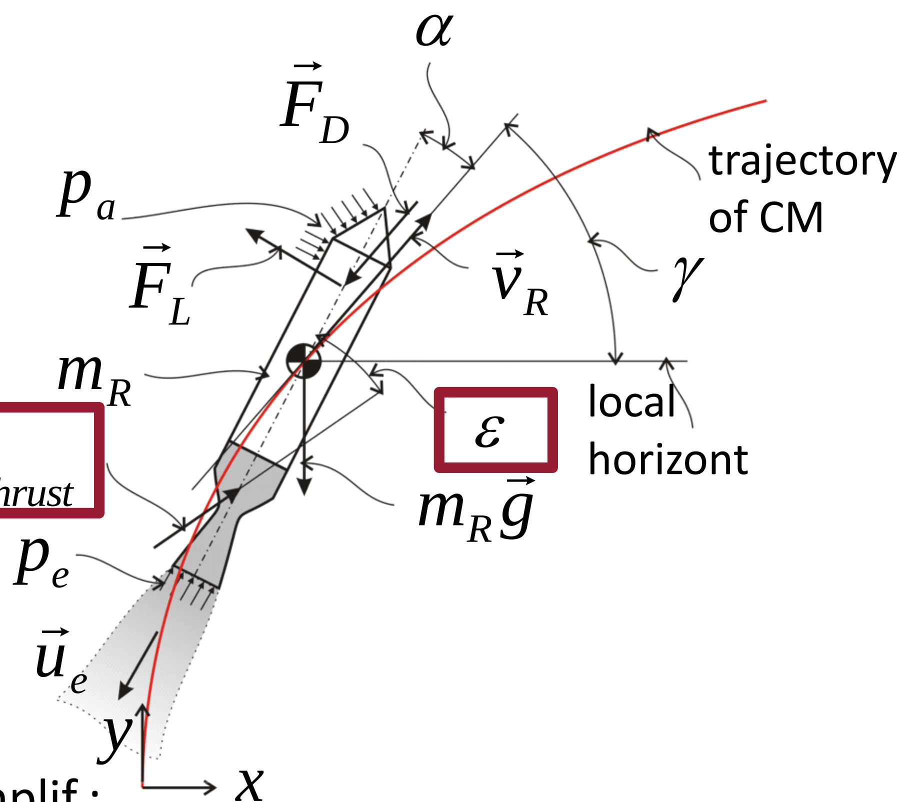
Three forces act on rocket at each instant:

1. gravitational force – applied at the CM $m_R\vec{g}$
2. aerodynamic force – applied at aerodynamic center and can be decomposed into:
   - drag force $\vec{F}_D$
   - lift force $\vec{F}_L$

they are function of vertical location and attitude of rocket

3. Performance of Rocket Vehicle

- Forces act on rocket



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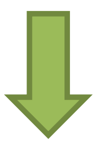
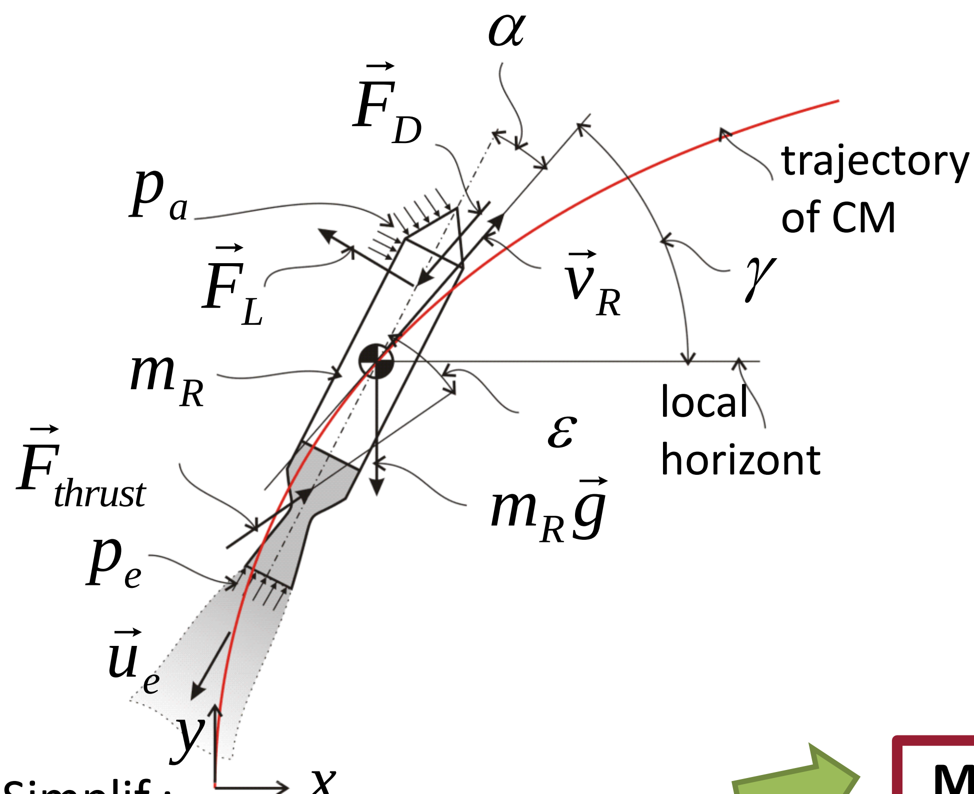


magnitude and direction can be controlled

3. Performance of Rocket Vehicle

- Forces act on rocket



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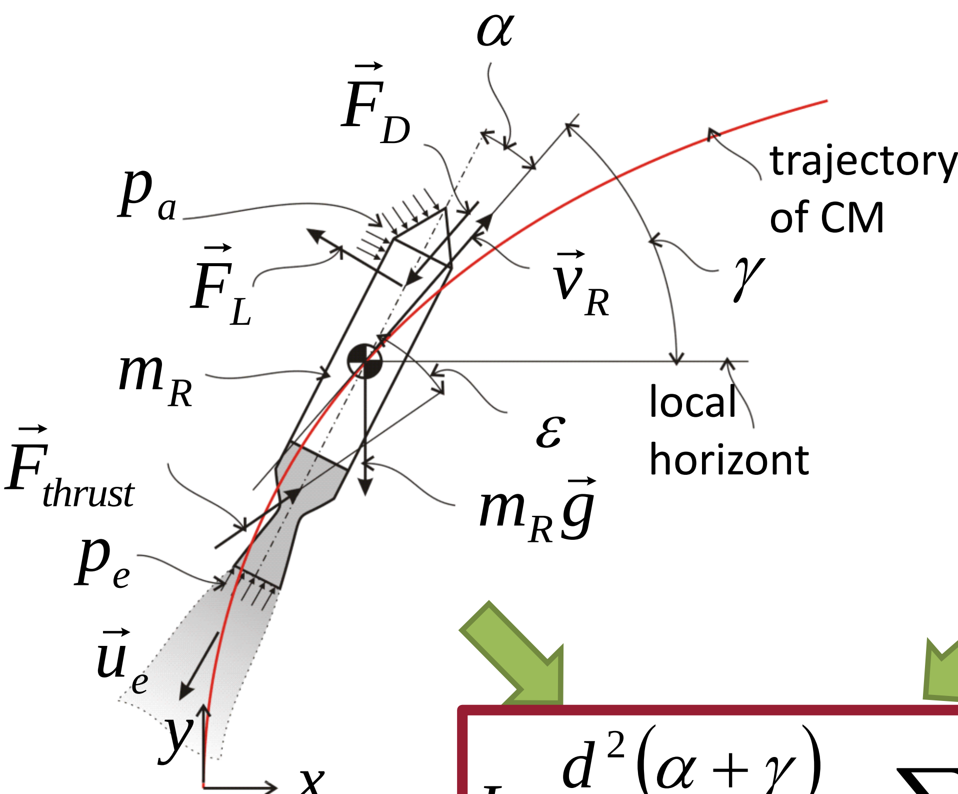
Motion of vehicle in 2D:
- translation motion of Center of Mass
- relative rotation motion around the CM

3. Performance of Rocket Vehicle

- Forces act on rocket



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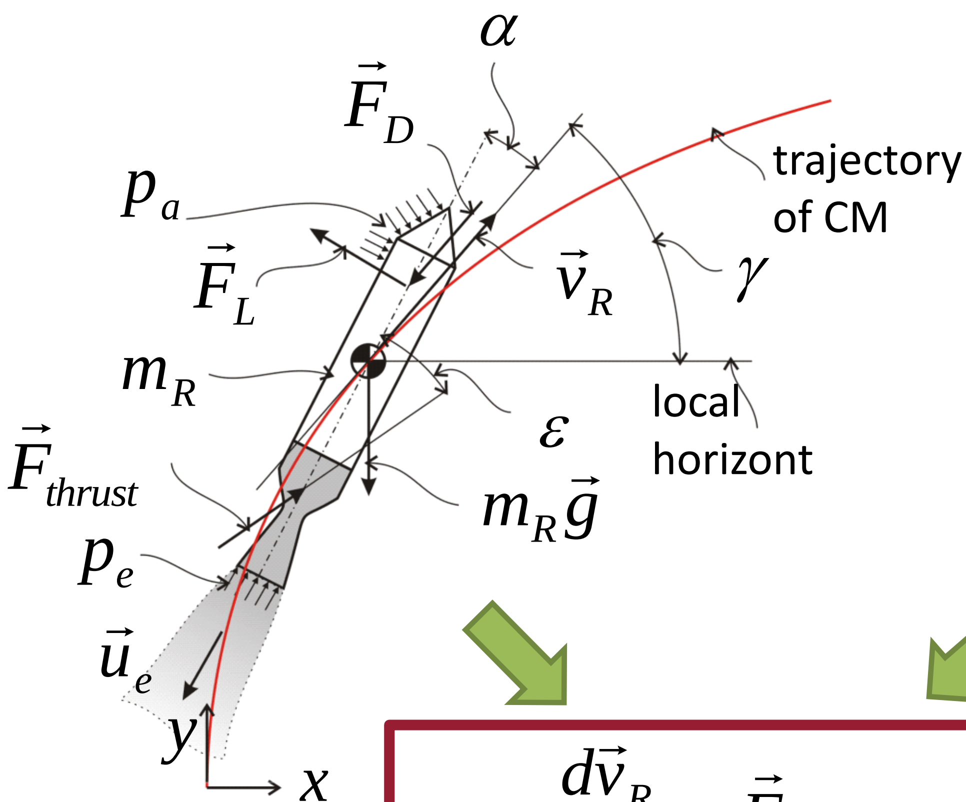
   - drag force $\vec{F}_D$
   - lift force $\vec{F}_L$
3. thrust force $\vec{F}_{thrust}$

$$I_R \frac{d^2(\alpha+\gamma)}{dt^2} = \sum_i M_{Fi}$$

dynamic equations: relative rotation motion around the CM

3. Performance of Rocket Vehicle

- Forces act on rocket



Three forces act on rocket at each instant:

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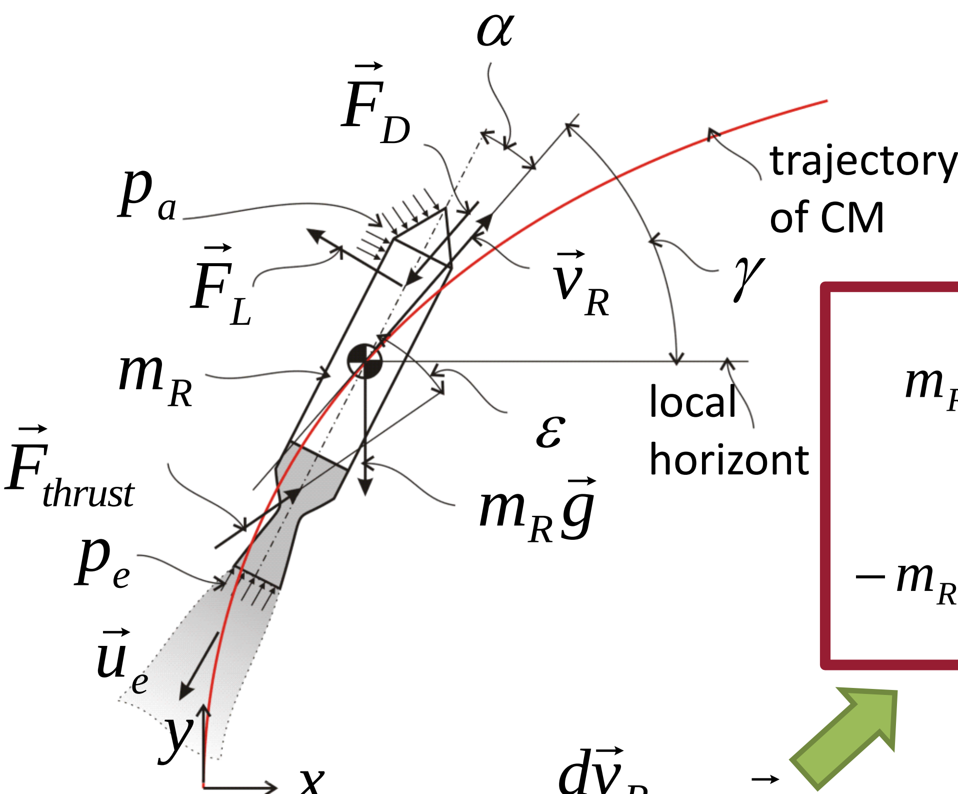
$$m_R \frac{d\vec{v}_R}{dt} = \vec{F}_{thrust} + m_R\vec{g} + \vec{F}_D + \vec{F}_L$$

dynamic equations: translation motion of the CM

3. Performance of Rocket Vehicle

Launch Flight Mechanics

- Forces act on rocket



tangent and normal decomposition

$$m_R \frac{dv_R}{dt} = F_{thrust} \cos(\varepsilon - \alpha) - m_R g \sin\gamma - F_D$$

$$-m_R v_R \frac{d\gamma}{dt} = F_{thrust} \sin(\varepsilon - \alpha) + m_R g \cos\gamma - F_L$$

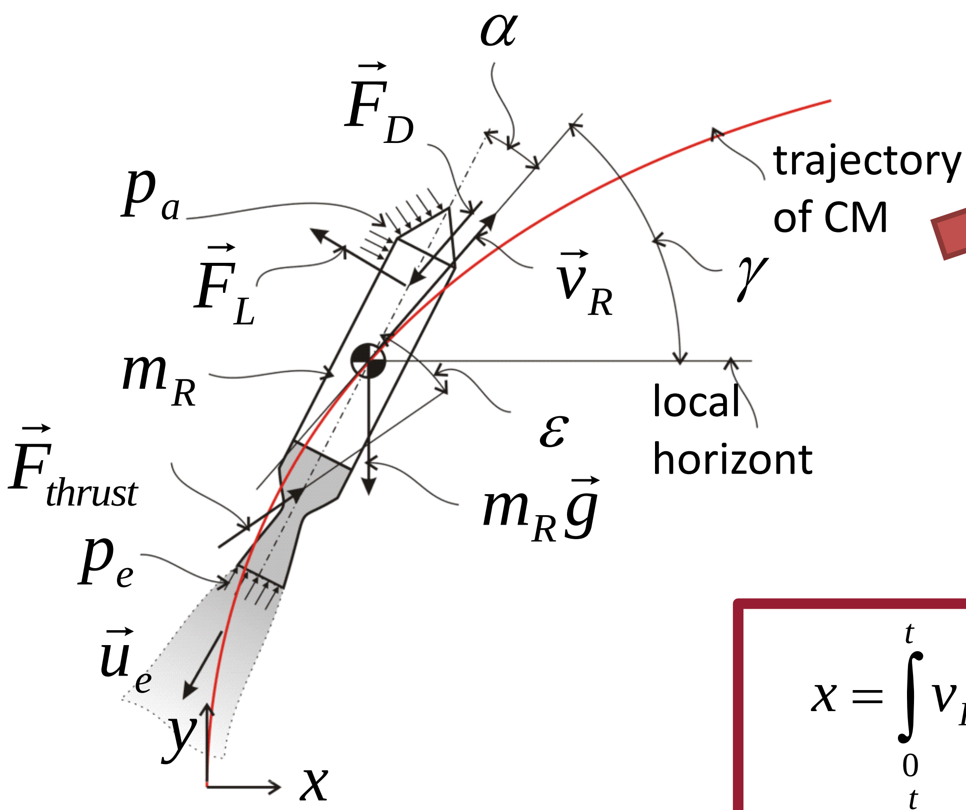
$$m_R \frac{d\vec{v}_R}{dt} = \vec{F}_{thrust} + m_R \vec{g} + \vec{F}_D + \vec{F}_L$$

dynamic equations: translation motion of the CM

3. Performance of Rocket Vehicle

Launch Flight Mechanics

- Forces act on rocket



Equations of rocket motion:
- dynamic equations
- kinematic equations
- equation of propellant mass flow

Numerical solution of system of ODE

$$x = \int_0^t v_R \cos\gamma \, dt$$

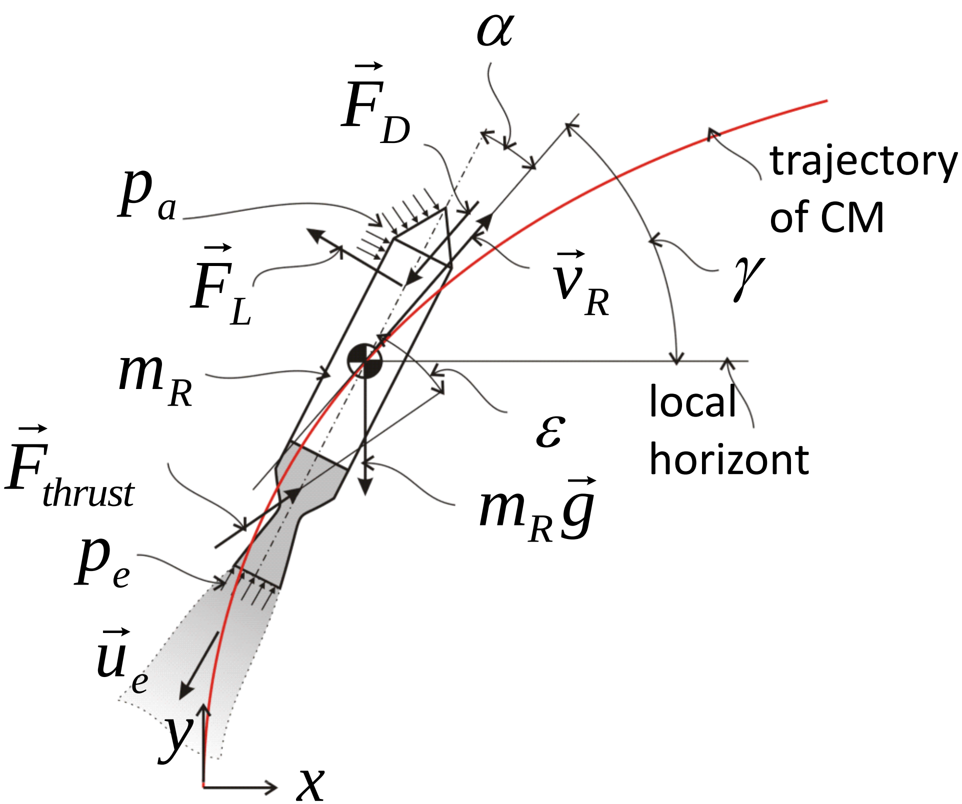
$$y = \int_0^t v_R \sin\gamma \, dt$$

kinematic equations: vertical and horizontal distance

3. Performance of Rocket Vehicle

Launch Flight Mechanics

- Forces act on rocket



Gravity turn:

- gravity turn trajectory – change of flight angle due to gravity
- only thrust and gravity is considered
- angle of attack is zero
- thrust is in axis of rocket

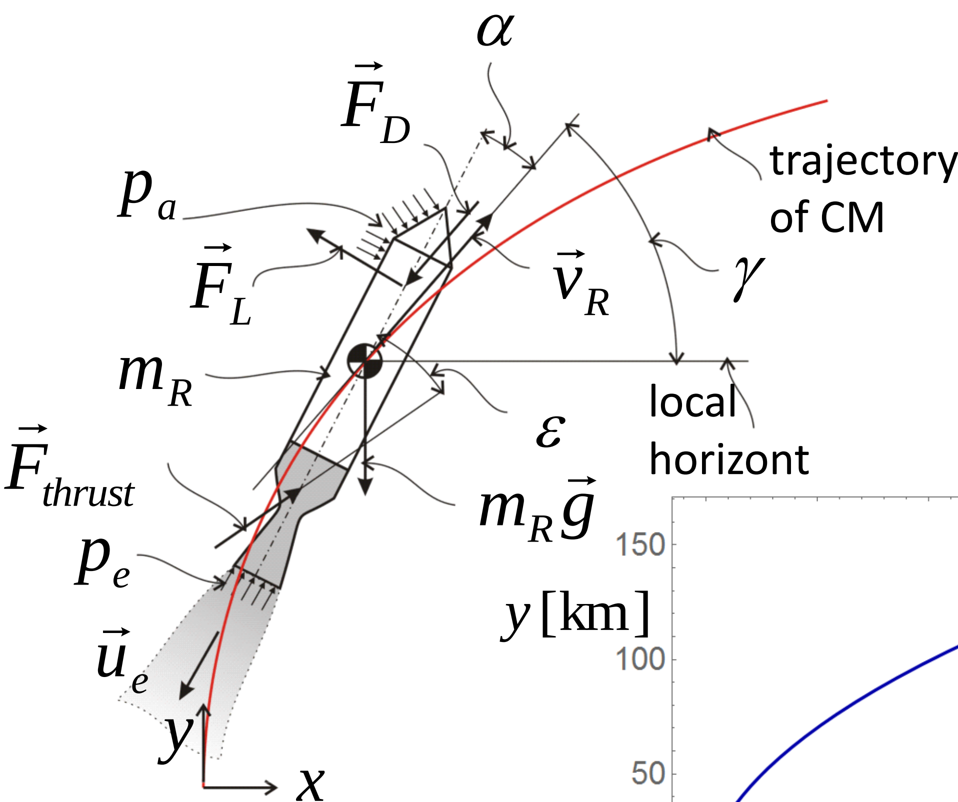
$$m_R \frac{dv_R}{dt} = F_{thrust} - m_R g \sin\gamma$$

$$v_R \frac{d\gamma}{dt} = -g \cos\gamma$$

3. Performance of Rocket Vehicle

Launch Flight Mechanics

- Forces act on rocket



Gravity turn:

- initial mass 90 t, propellant is 80% of mass with flow 250 kg/s effective velocity 4000 m/s
- in altitude 1 km, flight angle is changed to 89.85°

$$m_R \frac{dv_R}{dt} = F_{thrust} - m_R g \sin\gamma$$

$$v_R \frac{d\gamma}{dt} = -g\cos\gamma$$

$$\frac{dx}{dt} = v_R \cos\gamma$$

$$\frac{dy}{dt} = v_R \sin\gamma$$